International Economics, Lecture 1
Intertemporal Trade and the Current Account

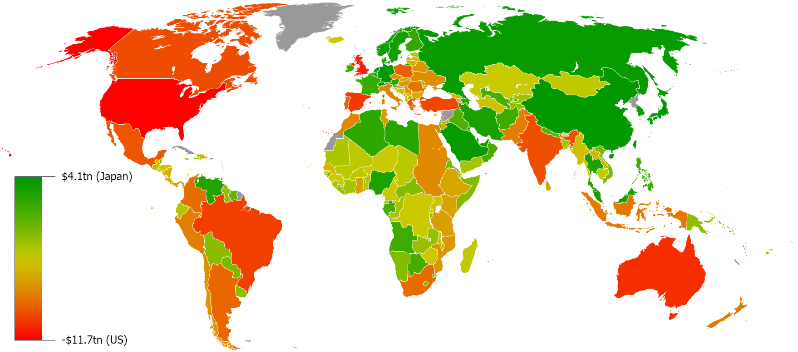
Dan Wales
University of Cambridge

London School of Economics
1 October, 2018

Course Overview (Michaelmas Term)

- Three broad topics in international macroeconomics.
  1. Real: Current account and international RBC.
  2. Nominal: Monetary policy and international pricing.
  3. Financial crises and the nominal exchange rate.
- Objectives:
  - Recent developments in international macroeconomics.
  - Develop tools and ideas for writing research papers.
- Deliverables:
  - Weekly problem sets (two marked at random during the term).
  - Exam (January).
  - Extended essay (should you choose so).

Cumulative Current Account Balances

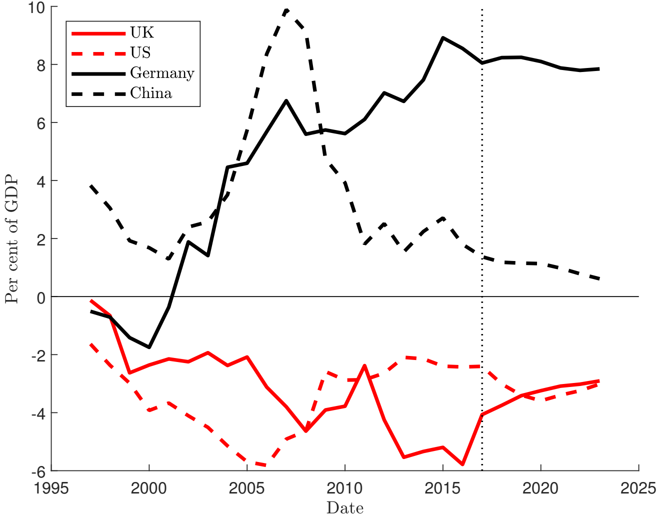


Notes: Data taken from IMF April 2018 WEO, (1980 - 2017). Methodology as in Schmitt-Grohé et al. (2016).

Why Focus on the Current Account?

- Indicator of **sustainability**.
  - Conventional wisdom: Persistent current account deficits above **5%** of GDP should be alarming.
    - Reversals are usually associated with **slower income growth**...
    - ...and large currency **depreciations**.
  - Can this country generate future trade surpluses to repay debt burden? Ultimately: is the country solvent?
- Indicator of **macroeconomic imbalances**.
  - Clearly highlights which countries are **reliant** on external financing, and imbalance between countries.
  - But **causality** is unclear.
- Highlight **intertemporal** trade.

Current Account Balances



Source: IMF April 2018 WEO.

Plan

- Basic Definitions
- Two Period Endowment Model (SOE)
- Two Period Production Model (SOE)
- Two Period Model (Two Country)
- Dynamics of the Current Account
- Stochastic Infinite Horizon Model
- Engel and Rogers (2006)*

Basic Definitions

- Balance of Payments.
  - Records transactions with the rest of the world.
  - Double entry bookkeeping, as each transaction enters twice.
  - Comprised of three separate accounts:

$$BoP = CA + KA - FA = 0.$$

- Current Account (CA):
  - Trade balance and net factor income from abroad.
    $ca_t \equiv nx_t + rb_{t-1}$.
- Capital Account (KA):
  - Net unilateral capital transfers from abroad (Small).
- Financial Account (-FA):
  - Net acquisition of foreign financial assets. $fa_t \equiv b_t - b_{t-1}$.

Three Current Account Relationships

- Definitional:

$$ca_t = nx_t + rb_{t-1} :$$

as the trade balance and net factor income from abroad.

- Change in net foreign assets:

$$ca_t = fa_t = b_t - b_{t-1},$$

which uses the BoP relationship (**intertemporal approach**). Arises as unilateral payments in the capital account are small.

- Difference between savings and investment:

$$\begin{aligned} y_t &= c_t + i_t + g_t + nx_t, \\ y_t &= c_t + i_t + g_t + ca_t - rb_{t-1}, \\ ca_t &= \underbrace{rb_{t-1} + y_t - c_t - \tau_t}_{s^p} + \underbrace{\tau_t - g_t}_{s^g} - i_t, \end{aligned}$$

which uses the GDP and BoP identities.

Two Period Endowment Model (SOE)

- Small Open Economy (SOE) takes world interest rate, r, as given.
- Representative household maximises lifetime utility:

  $$U = u(c_1) + \beta u(c_2),$$

  where c_t represents real consumption, $\beta \in (0, 1)$ is the discount factor and the period utility function, $u(\cdot)$, obeys standard properties $[u'(\cdot) >, u''(\cdot) < 0]$.
- Subject to period-t budget constraints:

  $$b_t + c_t = y_t + (1 + r)b_{t-1},$$

  for $t = \{1, 2\}$ where $b_0 = b_2 = 0$ and endowments, y_t, are perishable.

Intertemporal Budget Constraint

- Write down the period-t constraints:

$$\begin{aligned} b_1 + c_1 &= y_1 + (1+r)b_0, \\ b_2 + c_2 &= y_2 + (1+r)b_1. \end{aligned}$$

- Recall $b_0 = b_2 = 0$:

$$\begin{aligned} b_1 + c_1 &= y_1, \\ c_2 &= y_2 + (1+r)b_1, \end{aligned}$$

- Eliminate b_1 to show that:

$$c_1 + \frac{c_2}{1+r} = y_1 + \frac{y_2}{1+r}.$$

- The present discounted value of lifetime consumption equals the present discounted value of lifetime income.

Solution

- Substitute the IBC to rewrite the household problem as:

$$\max_{c_1} u(c_1) + \beta u(y_2 + (1+r)(y_1 - c_1)).$$

- Differentiate to show the first order condition:

$$\frac{u'(c_1)}{\beta u'(c_2)} = 1 + r.$$

- The **marginal rate of substitution** between c_1 and c_2 is equal to the **relative price** of current consumption vis-a-vis future consumption.

Autarky vs Openness

- Under autarky household consumes income:

$$c_t = y_t,$$
$$b_t = 0.$$

- In a trading equilibrium, in general:

$$c_t \neq y_t,$$
$$b_t \neq 0.$$

- Therefore define autarky real interest rate as:

$$\frac{u'(y_1)}{\beta u'(y_2)} = 1 + r^{Aut}.$$

Suppose $r^{Aut} > r$

- Rearrange budget constraints:

$$c_1 = y_1 - b_1,$$
$$c_2 = y_2 + (1+r)b_1.$$

- Then, using the FOC of the household problem:

$$1 + r^{Aut} \equiv \frac{u'(y_1)}{\beta u'(y_2)} > \frac{u'(c_1)}{\beta u'(c_2)} = 1 + r.$$

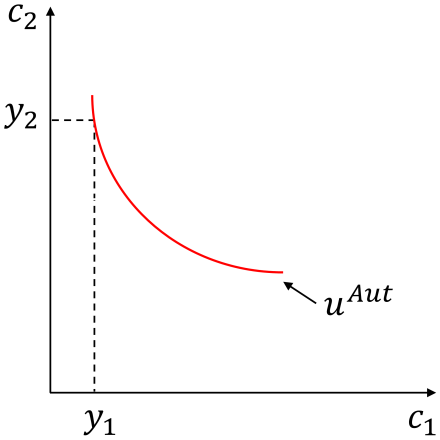
- Reveals:

$$\frac{u'(y_1)}{\beta u'(y_2)} > \frac{u'(y_1 - b_1)}{\beta u'(y_2 + (1+r)b_1)},$$

  such that $r^{Aut} > r$ [$r^{Aut} < r$] implies an initial current account deficit [surplus], as $b_1 < 0$.

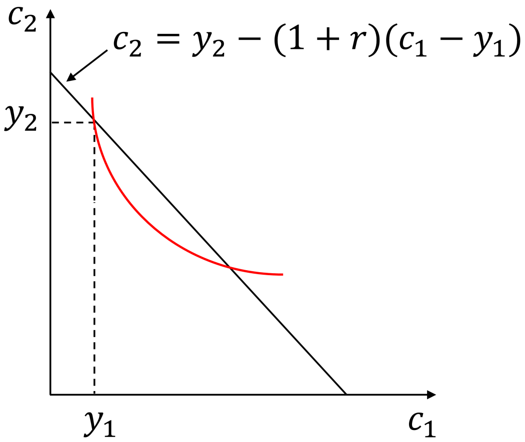
Graphical Representation - Autarky

- Initially consume endowments and receive some autarky level of utility, u^{Aut}.



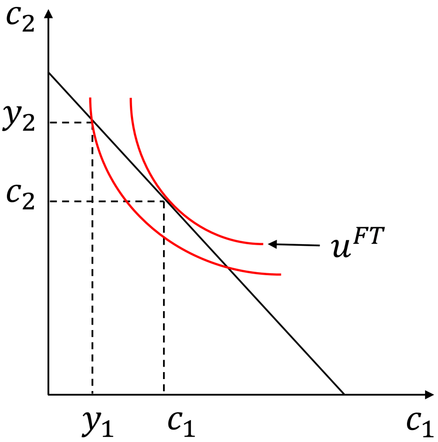
Graphical Representation - Introduce Trade

- Then, introduce possibility to trade using intertemporal budget constraint.



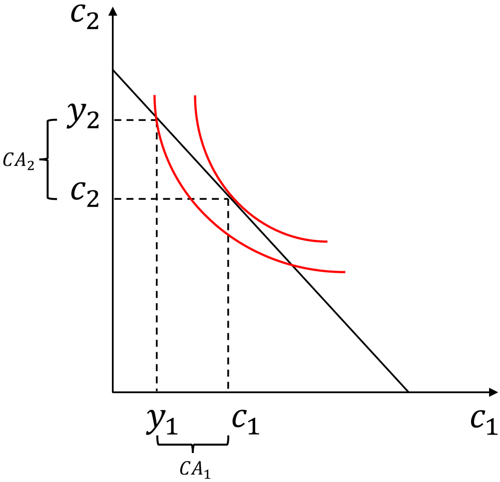
Graphical Representation - New Equilibrium

- Under free trade, the small open economy moves to the new equilibrium point on a higher indifference curve, u^{FT}.



Graphical Representation - Current Accounts

- Same diagram also shows the current account in each period. Borrowing today, repaying tomorrow.



Graphical Representation - What Happened?

- Initially the country was relatively well endowed with home endowment in the second period.
- At the prevailing world interest rate, r, they therefore wish to consume a little more today, and a little less tomorrow.
- Under free trade, they run a current account deficit today $CA_1 < 0$ such that $c_1 > y_1$...
- ... and a current account surplus tomorrow, $CA_2 > 0$, such that $c_2 < y_2$.
- Together this smooths consumption between periods, relative to the autarky case.
- Note: $nx_2 = -(1 + r)nx_1$, borrowing is repaid with interest.

Income Shocks and the Current Account

- Consider the model just described and assume $(1+r)\beta = 1$:

$$\frac{u'(c_1)}{\beta u'(c_2)} = 1 + r \rightarrow c_1 = c_2,$$

  and we therefore have **perfect consumption smoothing**.

- Use the IBC to find consumption:

$$c_1 = c_2 = \frac{(1+r)y_1 + y_2}{2+r}.$$

- Use the period budget constraint to solve for current account:

$$b_1 = y_1 - c_1 = \frac{y_1 - y_2}{2+r}.$$

- Also assume $y_1 = y_2$ such that, initially, $b_1 = 0$.

Permanent Shock

- Permanent increase in income: $\Delta y_1 = \Delta y_2 > 0$.
- Consumption in both periods **increases** to match the new level of discounted life-time income:

$$\Delta c_1 = \Delta c_2 = \frac{(1+r)\Delta y_1 + \Delta y_2}{2+r} > 0.$$

- **No change** to the current account, which is still balanced:

$$\Delta b_1 = \Delta y_1 - \Delta c_1 = \frac{\Delta y_1 - \Delta y_2}{2+r} = 0.$$

Temporary Shock

- Now consider a temporary increase in income: $\Delta y_1 > 0$ but $\Delta y_2 = 0$.
- Again, consumption in both periods increases to match the new level of discounted life-time income:

$$\Delta c_1 = \Delta c_2 = \frac{(1+r)\Delta y_1}{2+r} > 0,$$

  due to the change in y_1 only.
- However now the current account moves to **surplus**, as some of the temporary windfall is saved to **smooth consumption** over time:

$$\Delta b_1 = \Delta y_1 - \Delta c_1 = \frac{\Delta y_1}{2+r} > 0.$$

Intuition

- "If you lose your lunch money one day, it's not a problem. You simply borrow from a friend. Next time, you pay for his lunch. However if your parents cut your monthly allowance, you will have to change spending plans accordingly" [SGU, forthcoming].
- It is hard to identify whether a change in income is temporary or permanent. One exception to this are natural disasters.

Two Period Production Model (SOE)

- Now introduce production and government spending to the previous model. Each period, government purchases satisfy:

  $$g_t = \tau_t,$$

  where g_t is government spending and τ_t represent lump-sum taxes, faced by households.

- Production technology follows:

  $$y_t = A_t F(k_t),$$

  where $F(\cdot)$ obeys standard properties.
  $[F(0) = 0, F'(\cdot) >, F''(\cdot) < 0]$.

- Capital accumulation:

  $$k_{t+1} = k_t + i_t,$$

  where there is no depreciation, k_1 is given and investment may be negative.

New Intertemporal Budget Constraint

- The series of period-t budget constraints now become:

$$b_t + c_t = y_t + (1 + r)b_{t-1} - i_t - \tau_t,$$

  where τ_t represent lump-sum taxes and i_t is investment.

- Again, eliminate b_1 and set $b_0 = b_2 = 0$ to show that:

$$c_1 + i_1 + \frac{c_2 + i_2}{1 + r} = y_1 - \tau_1 + \frac{y_2 - \tau_2}{1 + r}.$$

- PDV of lifetime expenditure on consumption and investment equals the PDV of lifetime income, after tax.

- To move to equilibrium notice:
  - $k_3 = 0 \rightarrow i_2 = -k_2$ and $i_1 = k_2 - k_1$, with k_1 given.
  - Taxation is specified exogenously as government consumption.
  - Production functions: $y_t = A_t F(k_t)$.

Optimality Conditions

- Combine and rearrange for c_2:

$$c_2 = (1+r)\Big[A_1 F(k_1) - g_1 - c_1 - k_2 + k_1\Big] + A_2 F(k_2) - g_2 + k_2,$$

- Use in optimisation problem:

$$\max_{c_1,k_2} u(c_1)+\beta u\Big((1+r)(A_1 F(k_1)-g_1-c_1-k_2+k_1)+A_2 F(k_2)-g_2+k_2\Big),$$

- FOCs:

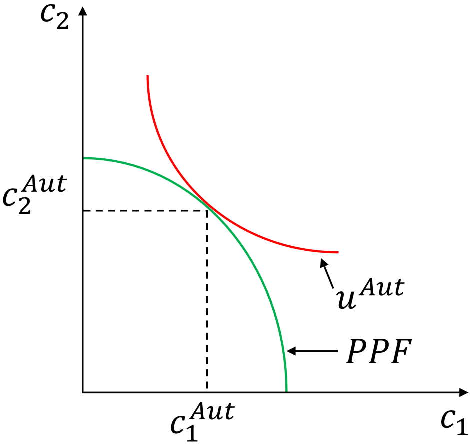
$$u'(c_1) = \beta(1+r)u'(c_2), \qquad \text{(wrt } c_1\text{)}$$
$$A_2 F'(k_2) = r. \qquad \text{(wrt } k_2\text{)}$$

- **Separation** of savings and investment decisions (k_2 independent of u).

- **No crowing out** (k_2 independent of g_t).

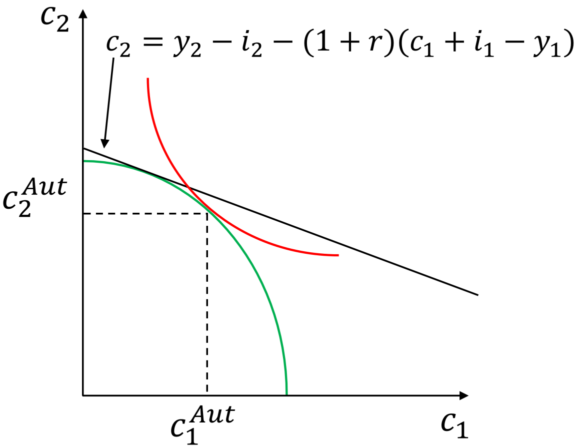
Graphical Representation - Autarky

- Initially consume endowments and receive some autarky level of utility, u^{Aut}. IC tangent to PPF.



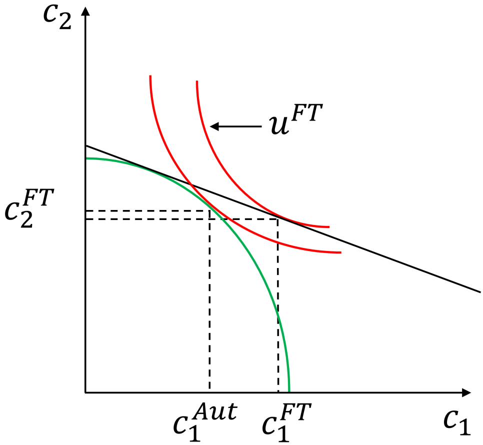
Graphical Representation - Introduce Trade

- Then, introduce possibility to trade using intertemporal budget constraint. BC is tangent to PPF.



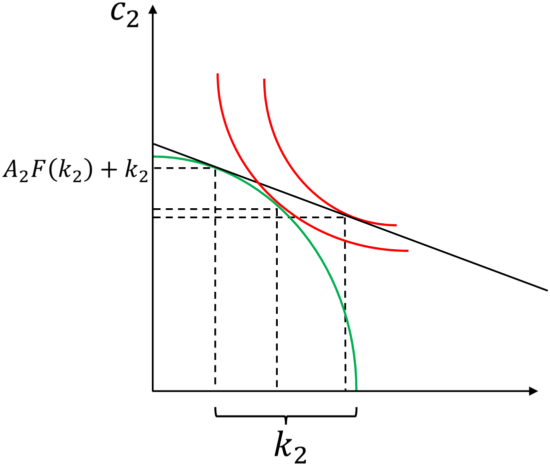
Graphical Representation - New Equilibrium

- Under free trade, households/consumption move(s) to the new equilibrium point on a higher indifference curve, u^{FT}.



Graphical Representation - Current Accounts

- Firms/production move(s) to a new equilibrium point. The level of capital, k_2, may be read directly from the graph.



Graphical Representation - What Happened?

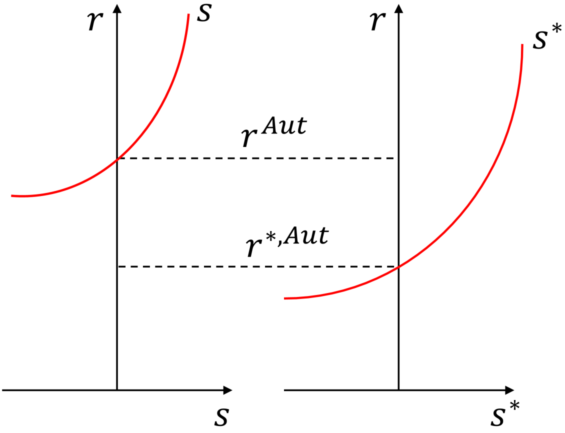
- Two distinct procedures:
  - In the first investment decisions (capital) are made to maximise the possible size of the economic pie for the Home country, given international prices.
  - In the second, given this level of income, international prices and the household discount factor, households choose consumption to maximise utility.

Two Period Model (Two Country)

- Two countries: Home and Foreign (denoted by $*$).
- Return to endowment setting. Global equilibrium now requires:
$$y_t + y_t^* = c_t + c_t^*.$$
- May be rewritten in terms of household savings $s_t \equiv y_t - c_t$:
$$s_t + s_t^* = 0.$$
- Or in terms of the current account (savings minus investment, but no investment here):
$$CA_t + CA_t^* = 0.$$
- The main change: world interest rate is now **endogenous**, ensuring equilibrium in the global asset market.

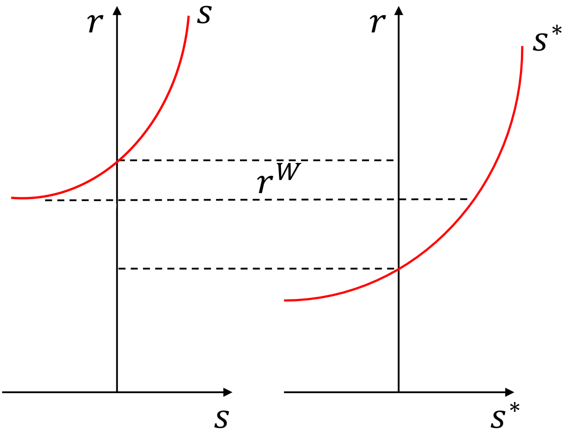
Graphical Representation - Metzler Diagram (Endowment)

- The savings schedule for each country may be plotted. Assume different autarky equilibrium real interest rates.



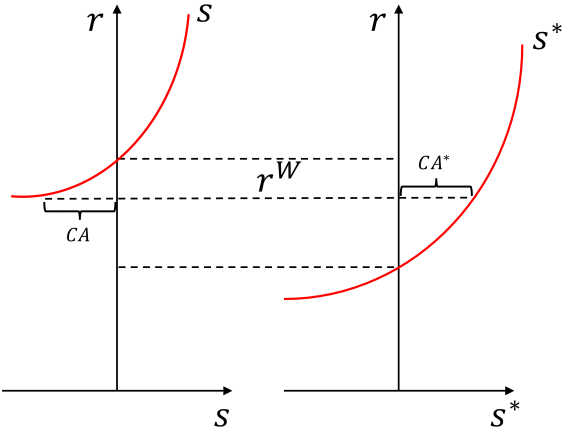
Graphical Representation - Free Trade I (Endowment)

- To balance the global level of household savings, the interest rate must fall for Home and rise for Foreign.



Graphical Representation - Free Trade II (Endowment)

- Indeed, we may quantify the precise level of r by considering the equilibrium condition $CA_t = -CA_t^*$.



Graphical Representation - What Happened?

- Under autarky for each country we define r^{Aut} and $r^{*,Aut}$ such that:

$$s_t(r_t^{Aut}) = 0,$$
$$s_t^*(r_t^{*,Aut}) = 0.$$

  - We **assumed** the parameterisation gave $r^{Aut} > r^{*,Aut}$.
- We must have that $r^{Aut} > r^W > r^{*,Aut}$, else:
  - $r^W > r^{Aut} > r^{*,Aut} \rightarrow CA_t > 0$ and $CA_t^* > 0 \rightarrow CA_t + CA_t^* > 0$.
  - $r^{Aut} > r^{*,Aut} > r^W \rightarrow CA_t < 0$ and $CA_t^* < 0 \rightarrow CA_t + CA_t^* < 0$.
- Therefore we know that when $r^{Aut} > r^W > r^{*,Aut}$:
  - $CA_t < 0$, while $CA_t^* > 0$.
  - Precise level given by $CA_t + CA_t^* = 0$.

Move to a Production Economy

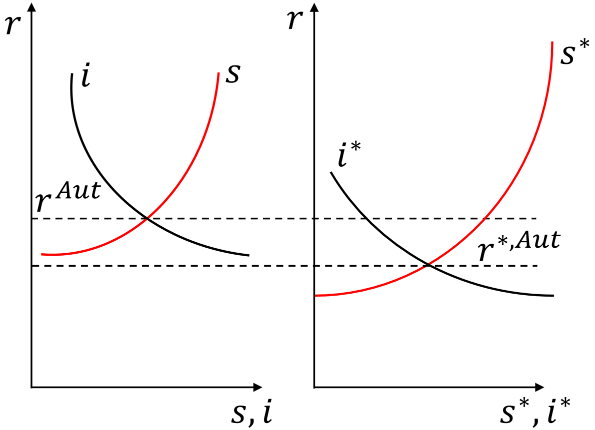
- The analysis for the two country case may be extended for an economy with **production**.
- Notice, in this case, current accounts are given as the **balance between household savings and investment**, $CA_t = s_t - i_t$, such that the real interest rate will be determined by:

$$s_t + s_t^* = i_t + i_t^*,$$
$$CA_t + CA_t^* = 0.$$

- For both endowment and production economies a shock to one country will propagate internationally to the other through changes in the real interest rate.

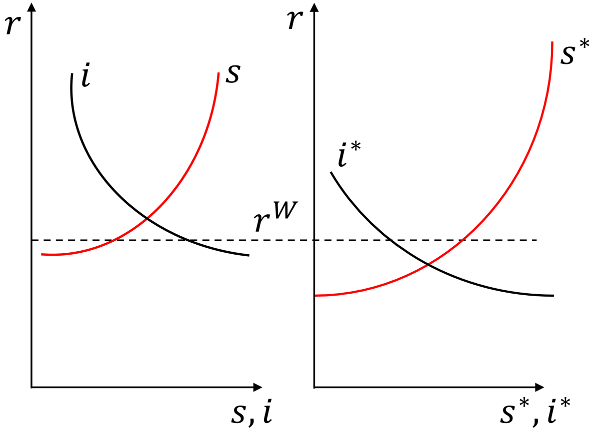
Graphical Representation - Metzler Diagram (Production)

- Savings and investment schedules are plotted for each country. Assume different autarky equilibrium real interest rates.



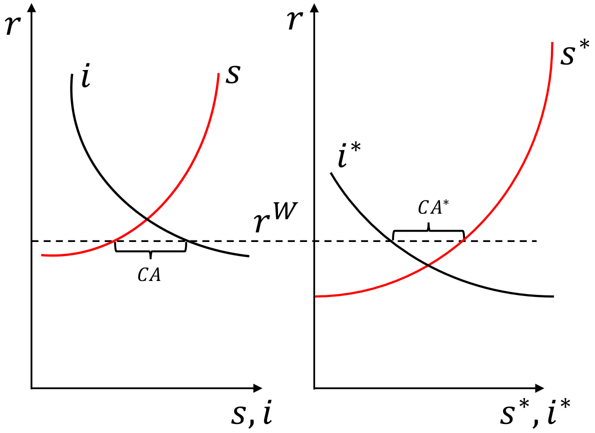
Graphical Representation - Free Trade I (Production)

- To balance the global level of savings and investment, the real interest rate must fall for Home and rise for Foreign.



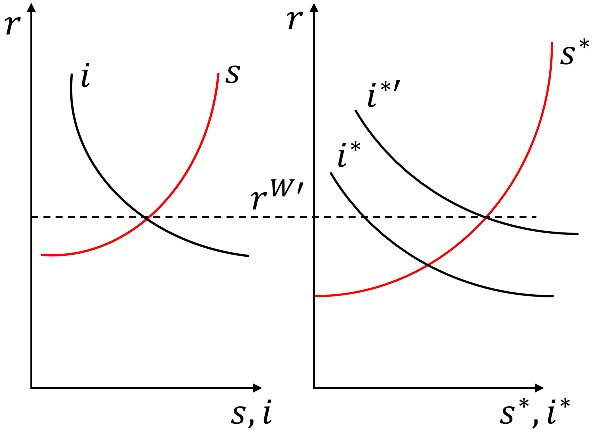
Graphical Representation - Free Trade II (Production)

- Again, we may quantify the precise level of r by considering the equilibrium condition $CA_t = -CA_t^*$.



Graphical Representation - Foreign Investment Shock

- The shock shifts the Foreign investment curve to the right, both countries affected. In one case both CA's close.



Dynamics of the Current Account

- Move from a 2-period setting to infinite horizon. Can do this in two steps:
  - Set a finite horizon, T.
  - Investigate properties as $T \to \infty$.
- Initially extend to a T-period model:
  - Representative household maximises lifetime utility:

$$U_t = u(c_t) + \beta u(c_{t+1}) + ... + \beta^T u(c_{t+T}) = \sum_{s=t}^{t+T} \beta^{s-t} u(c_s).$$

  - Subject to a series of period-t budget constraints:

$$\begin{aligned} b_t + c_t + i_t + g_t &= y_t + (1+r)b_{t-1}, \\ b_{t+1} + c_{t+1} + i_{t+1} + g_{t+1} &= y_{t+1} + (1+r)b_t, \\ &\cdots \\ b_{t+T} + c_{t+T} + i_{t+T} + g_{t+T} &= y_{t+T} + (1+r)b_{t+T-1}. \end{aligned}$$

  - Assume r is constant over time.

Finite Horizon Intertemporal Budget Constraint I

- As before rewrite the budget constraints in terms of NFA:

$$
\begin{aligned}
b_t &= y_t - [c_t + i_t + g_t] + (1+r)b_{t-1}, \\
\frac{b_{t+1}}{1+r} &= \frac{y_{t+1} - [c_{t+1} + i_{t+1} + g_{t+1}]}{1+r} + b_t, \\
&\dots \\
\frac{b_{t+T}}{1+r} &= \frac{y_{t+T} - [c_{t+T} + i_{t+T} + g_{t+T}]}{1+r} + b_{t+T-1}.
\end{aligned}
$$

- Iteratively substitute out to obtain:

$$
\frac{b_{t+T}}{(1+r)^T} = (1+r)b_{t-1} + \sum_{s=t}^{t+T} \frac{y_s - [c_s + i_s + g_s]}{(1+r)^{s-t}}.
$$

Finite Horizon Intertemporal Budget Constraint II

- Our usual assumption of $b_t = 0$ and argument that $b_{t+T} = 0$ will still apply.
- May therefore rewrite the intertemporal budget constraint as:

$$\sum_{s=t}^{t+T} \frac{y_s}{(1+r)^{s-t}} = \sum_{s=t}^{t+T} \frac{c_s + i_s + g_s}{(1+r)^{s-t}}.$$

  Again, it is clear that the PDV of lifetime income will be used to cover the PDV of expenditure on consumption, investment and by the government.
- Could stop here and discuss how this extension to a longer time horizon affects the model. A useful discussion in OR. Instead we proceed directly to the infinite horizon case.

Infinite Horizon Model

- Now allow $T \to \infty$.
- Representative household maximises lifetime utility:

$$U(c_t) = \sum_{s=t}^{\infty} \beta^{s-t} u(c_s).$$

- Subject to the intertemporal budget constraint:

$$\sum_{s=t}^{\infty} \frac{c_s + i_s + g_s}{(1+r)^{s-t}} + \lim_{T \to \infty} \frac{b_{t+T}}{(1+r)^T} = (1+r)b_{t-1} + \sum_{s=t}^{\infty} \frac{y_s}{(1+r)^{s-t}}.$$

Transversality Condition

- We will assume the terminal **transversality condition** (TVC):

$$\lim_{T\to\infty} \frac{b_{t+T}}{(1+r)^T} = 0.$$

- This is the result of two distinct notions:
  1. Optimality condition. With $u'(\cdot) > 0$ it will never be optimal for households to allow assets to grow more quickly that the real interest rate, such that:

  $$\lim_{T\to\infty} \frac{b_{t+T}}{(1+r)^T} \geq 0.$$

  2. No-Ponzi (Madoff) assumption:

  $$\lim_{T\to\infty} \frac{b_{t+T}}{(1+r)^T} \leq 0.$$

- Continue to normalise **initial** condition: $(1+r)b_{t-1} = 0$.

Optimality Conditions

- Take FOCs of the infinite horizon problem, subject to initial and terminal conditions:

$$u'(c_t) = \beta(1+r)u'(c_{t+1}), \quad \text{(wrt } c_t\text{, Euler Equation)}$$
$$A_{t+1}F'(k_{t+1}) = r. \quad \text{(wrt } k_{t+1}\text{)}$$

- Will hold in every time period.

Analytical Solution for Consumption Profile?

- In general, no. Three special cases:
  1. $\beta = \frac{1}{1+r}$, such that households **fully consumption smooth**.
  2. Iso-elastic utility function (log as special case).
  3. Stochastic income, with $\beta = \frac{1}{1+r}$ and quadratic utility.

Permanent Level / Annuity Values

- Consider a constant, $\tilde{x}_t$:

$$\sum_{s=t}^{\infty} \frac{\tilde{x}_t}{(1+r)^{s-t}} = \sum_{s=t}^{\infty} \frac{x_s}{(1+r)^{s-t}},$$

such that the NPV of this constant, $\tilde{x}_t$, is equal to the NPV of the steam ov the variable x_t. Hence:

$$\frac{\tilde{x}_t}{1-\frac{1}{1+r}} = \sum_{s=t}^{\infty} \frac{x_s}{(1+r)^{s-t}},$$

$$\tilde{x}_t = \frac{r}{1+r} \sum_{s=t}^{\infty} \frac{x_s}{(1+r)^{s-t}},$$

and $\tilde{x}_t$ is said to be the **annuity value** of the series x_t.

Special Case 1, $\beta = \frac{1}{1+r}$

- Assume $\beta = \frac{1}{1+r}$ and rewrite the Euler equation as:

$$u'(c_t) = \beta(1+r)u'(c_{t+1}), \rightarrow u'(c_t) = u'(c_{t+1}),$$

such that we have **full consumption smoothing**, $c_t = c_{t+1}$.

- Define household wealth, $\mathcal{W}_t$, as:

$$\mathcal{W}_t = (1+r)b_{t-1} + \sum_{s=t}^{\infty} \frac{y_s - [i_s + g_s]}{(1+r)^{s-t}},$$
$$= (1+r)b_{t-1} + \frac{1+r}{r}\left[\tilde{y}_t - \tilde{i}_t - \tilde{g}_t\right].$$

- But, using IBC and assumption consumption is shown to be a constant fraction of lifetime wealth, $\mathcal{W}_t$:

$$\mathcal{W}_t = \sum_{s=t}^{\infty} \frac{c_s}{(1+r)^{s-t}} = \frac{1+r}{r} c_t.$$

Special Case 1, $\beta = \frac{1}{1+r}$ ctd.

- Using these results we observe:

$$c_t = r \cdot b_{t-1} + \left[\tilde{y}_t - \tilde{i}_t - \tilde{g}_t\right],$$

such that consumption only responds to changes in the value of **permanent income**.

- Turing to the current account:

$$ca_t = r \cdot b_{t-1} + y_t - c_t - i_t - g_t,$$
$$ca_t = (y_t - \tilde{y}_t) - (i_t - \tilde{i}_t) - (g_t - \tilde{g}_t).$$

- Changes in the current account arise due to **temporary deviations** in income, investment and government spending from their permanent levels. Call this the **fundamental equation of the current account**.

Special Case 2, iso-elastic utility

- Assume:

$$u(c_t) = \frac{c^{1-1/\sigma}}{1-1/\sigma},$$

with $\sigma > 0$, and hence $u'(c_t) = c_t^{-1/\sigma}$.

- Euler equation may be rewritten as:

$$u'(c_t) = \beta(1+r)u'(c_{t+1}), \rightarrow c_{t+1} = \beta^\sigma (1+r)^\sigma c_t.$$

- Substitute to find value of lifetime wealth as:[1]

$$\mathcal{W}_t = \sum_{s=t}^{\infty} \frac{c_s}{(1+r)^{s-t}} = \frac{c_t}{1-\beta^\sigma(1+r)^{\sigma-1}}.$$

[1] Assume $\beta^\sigma(1+r)^{\sigma-1} < 1$.

Special Case 2, iso-elastic utility ctd.

- Thus:

$$c_t = \frac{r+\theta}{1+r}\mathcal{W}_t,$$

where $\theta \equiv 1 - \beta^{\sigma}(1+r)^{\sigma}$.

- And hence, with a slight change to before:

$$ca_t = (y_t - \tilde{y}_t) - (i_t - \tilde{i}_t) - (g_t - \tilde{g}_t) - \frac{\theta}{1+r}\mathcal{W}_t.$$

- Similar intuition to before, with the addition of a tilt factor, θ:
  - Consumption still a **constant fraction** of PDV lifetime wealth.
  - Current account arises through:
    1. Deviations of variables from their permanent level.
    2. Difference in household preferences, compared to the market real interest rate (tilt factor). More impatient → consume more today → CA deficit today.

Special Case 2, log utility

- Log utility is a special case of the above, with $\sigma = 1$.
- In this case the Euler condition will become:

$$c_{t+1} = \beta(1+r)c_t.$$

- We may characterise consumption growth patterns using a relationship between β and $(1+r)$:
  - If $\beta > (1+r)$ then we have $c_{t+1} > c_t$ as household are relatively **patient** (compared to international financial markets).
  - If $\beta < (1+r)$ then we have $c_{t+1} < c_t$ as household are relatively **impatient**.
  - If $\beta = (1+r)$ then we have $c_{t+1} = c_t$ and we recover the **perfect consumption smoothing** profile of special case 1.
  - Intuition may also be given for a fixed β and considering changes in r.

Stochastic Infinite Horizon Model

- Introduce concepts allowing income to be **stochastic**:
  - Assume that y_t, i_t and g_t follow stochastic processes with $y_t \overset{\text{iid}}{\sim} \mathcal{N}(\mu_y, \sigma_y^2)$, $i_t \overset{\text{iid}}{\sim} \mathcal{N}(\mu_i, \sigma_i^2)$ and $g_t \overset{\text{iid}}{\sim} \mathcal{N}(\mu_g, \sigma_g^2)$.
  - Households have **rational expectations**:
    - Knows model's structure and shocks.
    - Forecast errors are uncorrelated, after conditioning on the available information set.
- Rewrite the household's optimisation problem as:

$$\max_{\{c_s\}_{s=t}^{\infty}} \mathbb{E}_t \sum_{t=1}^{\infty} \beta^{t-1} u(c_t),$$

$$\text{s.t.} \quad \mathbb{E}_t \sum_{s=t}^{\infty} \frac{c_s + i_s + g_s}{(1+r)^{s-t}} = (1+r)b_{t-1} + \mathbb{E}_t \sum_{s=t}^{\infty} \frac{y_s}{(1+r)^{s-t}},$$

$$\text{and} \quad \lim_{T\to\infty} \mathbb{E}_t \frac{b_{t+T}}{(1+r)^T} = 0.$$

Optimality Conditions In Stochastic Model

- Euler equation:

$$u'(c_t) = \beta(1 + r)\,\mathbb{E}_t[u'(c_{t+1})].$$

- Now "holds in expectation" and takes the uncertainty of income fluctuations into account.

Aside: Steady State Indeterminacy*

- A steady state arises when $x_t = x$, $\forall t, x$. (Long run).
- Three equations specify the equilibrium. In steady state these may be written as:

$$u'(c) = \beta(1+r)u'(c), \qquad \text{(EE)}$$
$$r = AF'(k), \qquad \text{(FOC, k)}$$
$$c + g = rb + AF(k). \qquad \text{(IBC)}$$

- EE only restricts the real interest rate, as $\beta(1+r) = 1$.
- From this, the FOC for capital pins down k.
- However, the IBC is then unable to distinguish between the level of consumption and international financial assets. The level of financial assets, b, is **indeterminate** and the steady state is compatible with any level of foreign assets.
- Schmitt-Grohé and Uribe (2003) document ways to solve this.

Special Case 3, Stochastic Income and Quadratic Utility

- Assume utility is quadratic:

$$u(c_t) = c_t - \frac{\alpha}{2}c_t^2,$$

with $\alpha > 0$ such that $u'(c_t) = 1 - \alpha c_t$.

- The Euler equation takes the form of a **random walk**[2]:

$$u'(c_t) = \beta(1+r)\,\mathbb{E}_t[u'(c_{t+1})] \rightarrow c_t = \mathbb{E}_t\, c_{t+1},$$

- From (abuse of) previous notation we have that:

$$\mathcal{W}_t = \frac{1+r}{r}c_t = (1+r)b_{t-1} + \frac{1+r}{r}\left[\tilde{y}_t - \tilde{i}_t - \tilde{g}_t\right],$$

where now $\tilde{x}_t \equiv \frac{r}{1+r}\sum_{s=t}^{\infty}\frac{\mathbb{E}_t\, x_s}{(1+r)^{s-t}}$.

[2] Classic reference is Hall (1978).

Special Case 3 ctd.

- With:

$$c_t = r \cdot b_{t-1} + \left[\tilde{y}_t - \tilde{i}_t - \tilde{g}_t\right],$$

we again have a version of the **fundamental equation of the current account**:

$$ca_t = (y_t - \tilde{y}_t) - (i_t - \tilde{i}_t) - (g_t - \tilde{g}_t).$$

where changes are now due to variables varying from their expected levels, and the variances of shocks are irrelevant for consumption decisions (**certainty equivalence**).

Example - Persistent Income I

- Suppose endowment model, with income following an AR(1) process (SGU 2.2):

$$y_t = (1-\rho)\mu_y + \rho y_{t-1} + \varepsilon_t,$$

with $\varepsilon_t \overset{\text{iid}}{\sim} \mathcal{N}(0, \sigma_\varepsilon^2)$ and $\rho \in (-1, 1)$, empirically $\rho > 0$.

- Can rewrite:

$$y_{t+s} - \mu_y = \rho(y_{t+s-1} - \mu_y) + \varepsilon_{t+s}.$$

- Implies:

$$\mathbb{E}_t[y_{t+s} - \mu_y] = \rho^s (y_t - \mu_y),$$

- Hence:

$$\tilde{y}_t = \frac{r}{1+r} \sum_{s=t}^{\infty} \frac{\mathbb{E}_t\, y_{t+s}}{(1+r)^{s-t}} = \mu_y + \frac{r}{1+r-\rho}(y_t - \mu_y).$$

Example - Persistent Income II

- Using $\tilde{y}_t$, given initial conditions y_{t-1}, b_{t-1} and an exogenous shock, ε_t, we may immediately write down the path for consumption and the current account:

$$c_t = r \cdot b_{t-1} + \tilde{y}_t = r \cdot b_{t-1} + \mu_y + \frac{r\rho(y_{t-1} - \mu_y) + r\varepsilon_t}{1 + r - \rho},$$

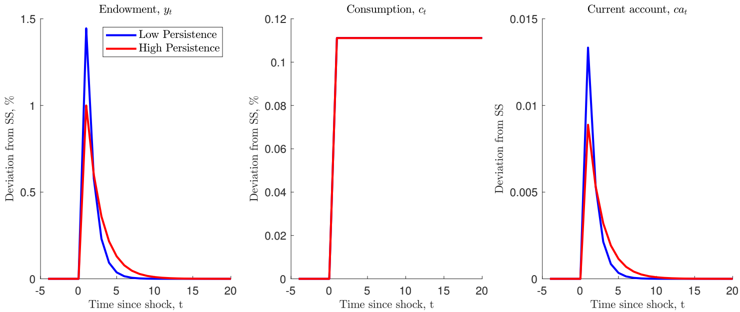
$$ca_t = y_t - \tilde{y}_t = \frac{\rho(1-\rho)(y_{t-1} - \mu_y) + (1-\rho)\varepsilon_t}{1 + r - \rho}.$$

- With **temporary** shocks $(0 < \rho < 1)$
  - Unexpected positive endowment shocks $(\varepsilon_t > 0)$ leads to $ca > 0$.
  - Want to **smooth consumption gains** over time.
- With **permanent** shocks $(\rho = 1)$
  - No effect on current account.
  - Income socks reflected one-to-one in consumption.
  - **Permanent income hypothesis** at work (Friedman).

Example - Persistent Income III

- Homework: Go to Matlab, generate the shock.

Impulse Response to Exogenous Income Shock



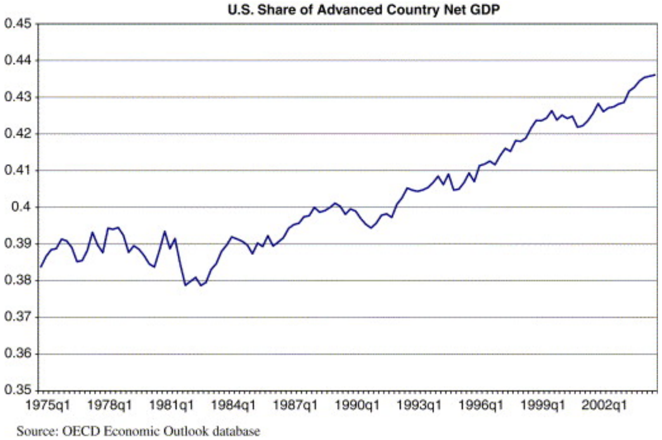
Notes: $\mu_y = 1$, $\rho_H = 0.6$, $\rho_L = 0.4$, $r = 5\%$. Calibrated for 1% income shock in highly persistant case. y_t and c_t shown as % deviation from SS.

Engel and Rogers (2006)*

- Question: Is the (large) US current account (deficit) an outcome of **optimising** behaviour?
- Their answer: Perhaps.
- Model: Two country current account model, tweaked to account for private forecasters expectations of future income growth.

US Share of AE GDP*

- Key observation: concurrent increase in US GDP as a share of total, and increase in CA deficit since 1980s.



Source: Engel and Rogers (2006).

Simple Two Country Model*

- Predictions:
  - Suppose $\mathbb{E}_t[\Delta y_{t+k}] > 0$ for some $k > 0$.
  - Then, optimising behaviour suggests home country should borrow now and repay later (when they have higher income). Hence CA deficit $ca_{t+k-s} < 0$ (for $0 < s < k$).
  - A low real interest rate would amplify this mechanism.
- Specifics:
  - Endowment economies, same log preferences, no government.
  - Equilibrium conditions (+ foreign):

$$c_{t+1} = \beta(1+r)c_t, \qquad \text{(EE)}$$

$$b_{t-1} = \mathbb{E}_t\left[\sum_{s=t}^{\infty} \frac{c_s - y_s}{(1+r)^{s-t}}\right], \qquad \text{(IBC)}$$

$$y_t^w = y_t + y_t^* = c_t + c_t^*. \qquad \text{(Resources)}$$

Use Fractions of World GDP*

- Combine and rewrite the above 3 equations as:

$$c_t = (1-\beta)\left((1+r)b_{t-1} + y_t^w\,\mathbb{E}_t\left[\sum_{s=t}^{\infty}\beta^{s-t}\gamma_s\right]\right),$$
$$c_t = (1+r)(1-\beta)b_{t-1} + y_t\Gamma_t/\gamma_t,$$

where $\gamma_s = \frac{Y_t}{Y_t^w}$ is the home country share of world GDP and $\Gamma_t = (1-\beta)\,\mathbb{E}_t\left[\sum_{s=t}^{\infty}\beta^{s-t}\gamma_s\right]$.

- The current account as a percentage of GDP becomes:

$$\frac{ca_t}{y_t} = \frac{b_t - b_{t-1}}{y_t} = \frac{y_t - c_t + rb_{t-1}}{y_t},$$
$$\frac{ca_t}{y_t} = 1 - \Gamma_t/\gamma_t - [1-\beta(1+r)]\frac{b_{t-1}}{y_t}.$$

Economic Mechanism*

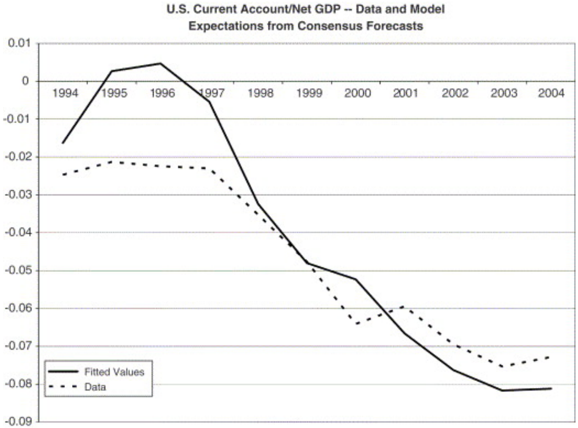
- Two key equations:

$$\frac{ca_t}{y_t} = 1 - \Gamma_t/\gamma_t - [1 - \beta(1+r)]\frac{b_{t-1}}{y_t},$$
$$\frac{c_t}{y_t} = (1+r)(1-\beta)\frac{b_{t-1}}{y_t} + \Gamma_t/\gamma_t.$$

- Assume, initially, $b_{t-1} = 0$ and that $\gamma_{t+k} \uparrow$ for some $k >> 0$. Then $c_t \uparrow$ and $ca_t \downarrow$.
- Note: The home country share of world output cannot grow indefinitely! The current account will rebalance at some point in the future.

Model Predictions*

- The model does well, after several "heroic assumptions" (see section 2.2) and using private forecasters expectations of US output growth to help dynamics.



Source: Engel and Rogers (2006).

Summary

- Introduced basic open economy concepts, and focus on current account.
- Develop both a two-period and infinite horizon model to describe current account dynamics.
- Three special cases to achieve an analytical solution.
- Application: US current account deficit in early 2000s (discussed findings in Engel and Rogers (2006)).

References I

Engel, C. and J. H. Rogers (2006). The U.S. Current Account Deficit and the Expected Share of World Output. *Journal of Monetary Economics 53*(5), 1063–1093.

Hall, R. E. (1978). Stochastic Implications of the Life Cycle-Permanent Income Hypothesis: Theory and Evidence. *Journal of Political Economy 86*(6), 971–87.

Schmitt-Grohé, S. and M. Uribe (2003). Closing Small Open Economy Models. *Journal of International Economics 61*, 163–185.

Schmitt-Grohé, S., M. Uribe, and M. Woodford (2016). *International Economics*. Princeton, NJ: Princeton University Press.