Part IIB - Economic Growth
Additional Questions

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The following questions were removed from the problem sets in 2020. They are still indicative of the style of question associated with the course. It may therefore be useful to prepare answers to these questions.

Problem Set 1

Question 1 *"The neoclassical growth model fits empirical data qualitatively, but not quantitatively."* Discuss this statement with reference to:

(a) Convergence of income per capita levels across countries over time.

(b) Evidence on interest rates and capital flows.

Problem Set 2

Question 1 *(Based on question 8 of the 2007 ECONOMICS TRIPOS Part IIB) Consider the following three sector model of the economy developed by Romer (1990). Competitive firms (indexed by i) in the final goods sector maximise profits by producing output, Y_i; by hiring labour $L_{Y,i}$ and a number of different capital goods, $x_{j,i}$, taking the wage w and rental prices p_j as given:*

$$\max_{Y_i, L_{Y,i}, x_{j,i}} Y_i - w L_{Y,i} - \sum_{j=1}^{A} p_j x_{j,i},$$

subject to production technology, which satisfies:

$$Y_i = L_{Y,i}^{1-\alpha} \sum_{j=1}^{A} x_{j,i}^{\alpha},$$

where $\alpha \in (0,1)$. A is the number of capital goods that are available to the final goods sector at any point in time. Note that the price of final output is normalised to 1 and capital goods depreciate fully after one period. Producers in the intermediate capital goods sector own the monopoly right (patent) to produce capital goods, x_j. Intermediate producers convert one unit of existing capital (rented at interest rate r) into intermediate capital goods. They maximise profits according to:

$$\max_{x_j} \pi_j = \max_{x_j} p_j(x_j)x_j - rx_j,$$

where $p_j(x_j)$ is the demand function for the capital good in the final goods sector. Scientists in the research sector produce new patents using labour, L_A, and existing technology, A:

$$\dot{A} = L_A A$$

Assume that labour markets clear, so that $L_Y + L_A = L = 1$.

(a) Give an economic interpretation of the production function of final output. Derive the first-order conditions characterising the wage w and rental price of intermediate capital goods p_j paid by each final goods producer.

(b) Solve for the optimal rental price p_j charged by intermediate firms producing capital goods x_j. Solve for profits π_j and aggregate output Y as a function of the aggregate capital stock $K = \sum_{j=1}^{A} x_j$. (Hint: each intermediate producer produces the same amount $x_j = x$). Provide a brief economic interpretation of the solutions.

(c) Using an arbitrage equation for returns in the research sector and the interest rate r, solve for the optimal price of a patent P_A. Give an intuitive explanation.

(d) Derive the equilibrium interest rate r as function of parameters and the output/capital ratio Y/K. Give an intuitive explanation of the result.

References

Romer, P. M. (1990). Endogenous Technological Change. *Journal of Political Economy 98*(5), 71 – 102.