Part IIB

Supervision 9 - Revision

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10 May 2021, University of Cambridge

This Class

- Sample Exam Paper 2020-2021.
- Beware own solutions which may contain typos.
  - Question 1: Sovereign Debt-Dynamics.
  - Question 2: Growth & Business Cycles.
  - Question 3: Business Cycles.
  - Question 4: International.
- Your Questions.
- Some Revision Tips.

Question 1: Sovereign Debt-Dynamics.

Question 1 - Set Up

- *Following the new coronavirus pandemic, many countries have experienced a sharp economic slowdown and many governments have engaged in expansionary fiscal policy leading to a large increase in the government budget deficit. Analyse the implications of these changes on the dynamics of the public debt-to-GDP ratio, and discuss whether they affect the possibility of sovereign debt crises.*

- Is this question on **multiple** topics or **just one**?

- Take in two parts. Discuss both **conventional** debt dynamics framework and then sovereign debt **crises** model.

Question 1 - Conventional Debt Dynamics

- Recall our accounting identity for the evolution of the debt-to-GDP ratio:

$$\Delta b = d + (r - g)b.$$

- Three forces. Mechanical effect of deficit (policy choice) and "forcing variables" (natural evolution).

- Steady state $\Delta b = 0$ arises at:

$$b^* = \frac{d}{g - r}.$$

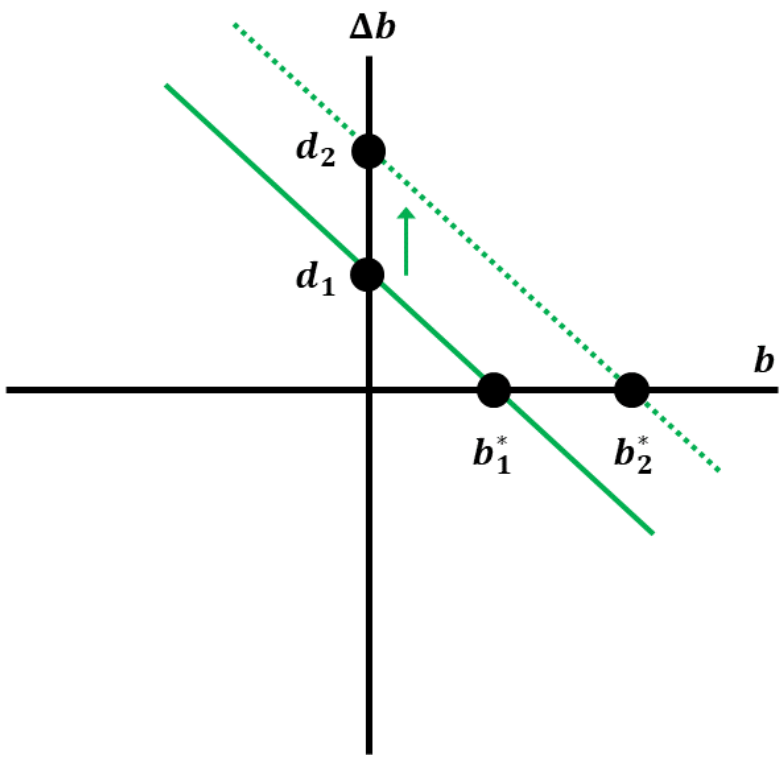
- This is positive in most countries, with $g > r$ and $d > 0$.

- COVID caused $g \downarrow$ and $d \uparrow$, hence debt accumulation with $b^* \uparrow$.

Question 1 - Transition to New Steady State I

- A higher government deficit, d, will increase the intercept in phase diagram, shifting the steady state to the right.

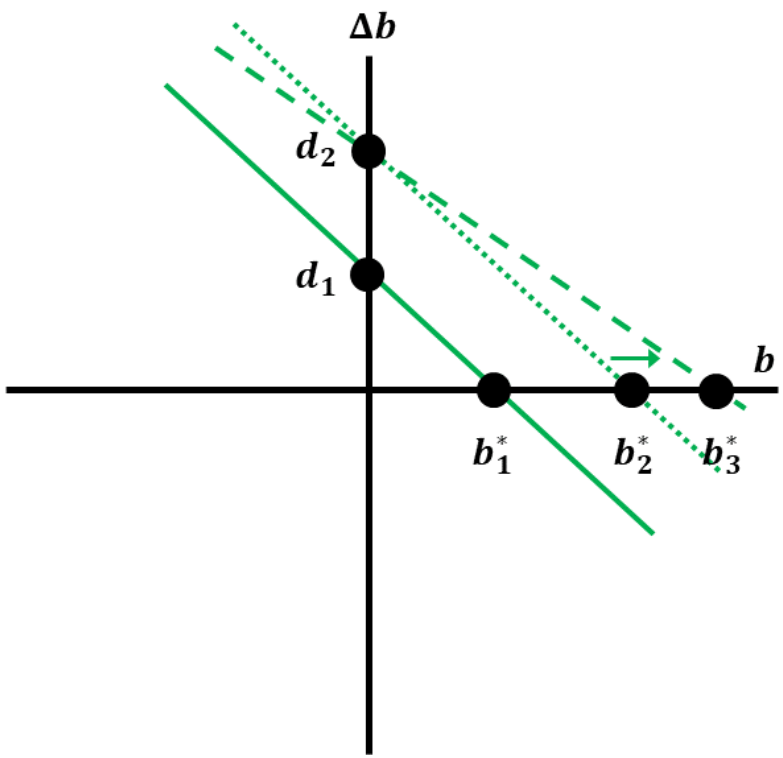
Phase Diagram and Time Path



Question 1 - Transition to New Steady State II

- Lower growth, g, reduces natural erosion of debt-to-GDP, lowering the slope in phase diagram, further increasing b^*.

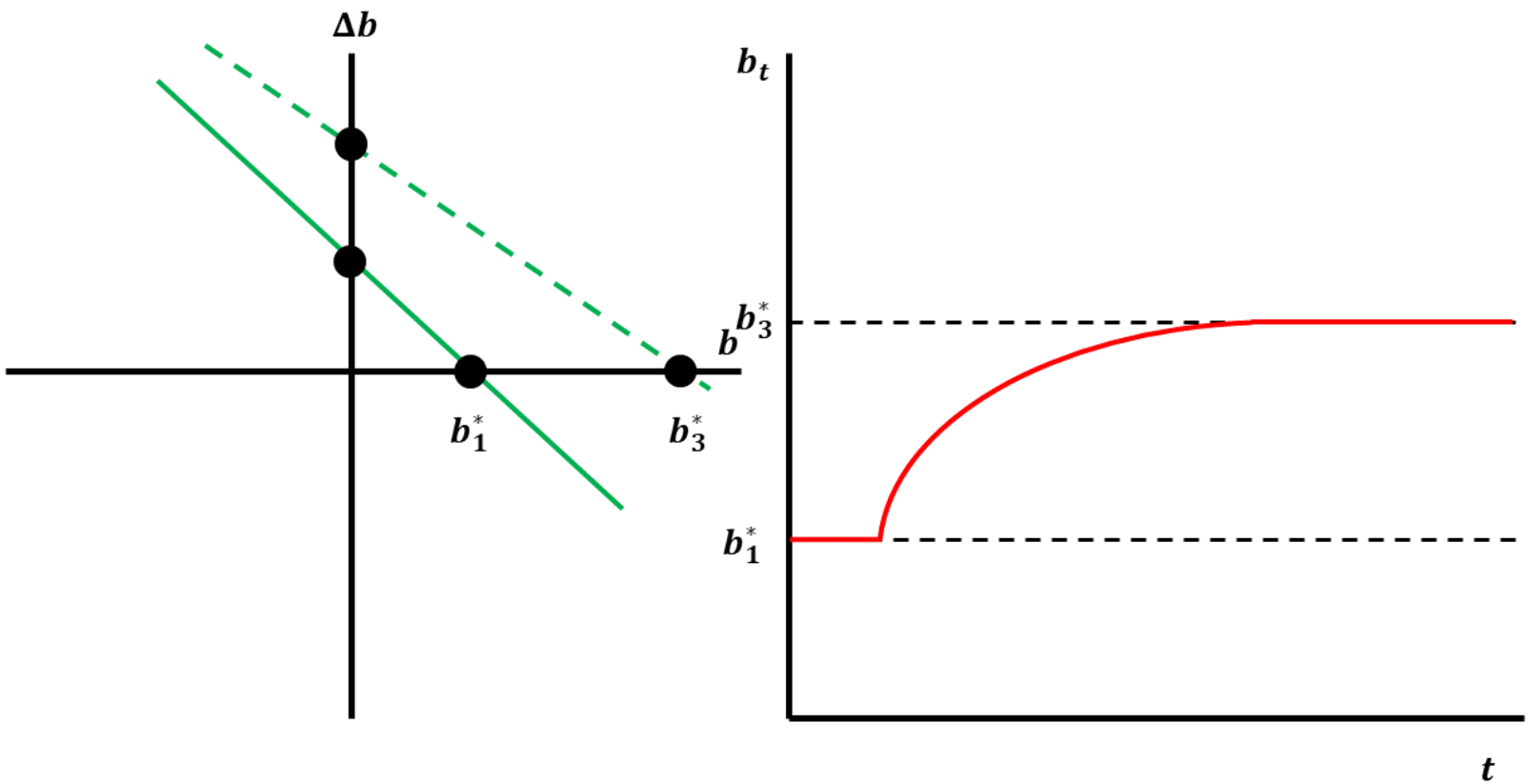
Phase Diagram and Time Path



Question 1 - Transition to New Steady State III

- Overall, initial rapid increase in debt followed by more gradual transition to new, higher, level of debt-to-GDP.

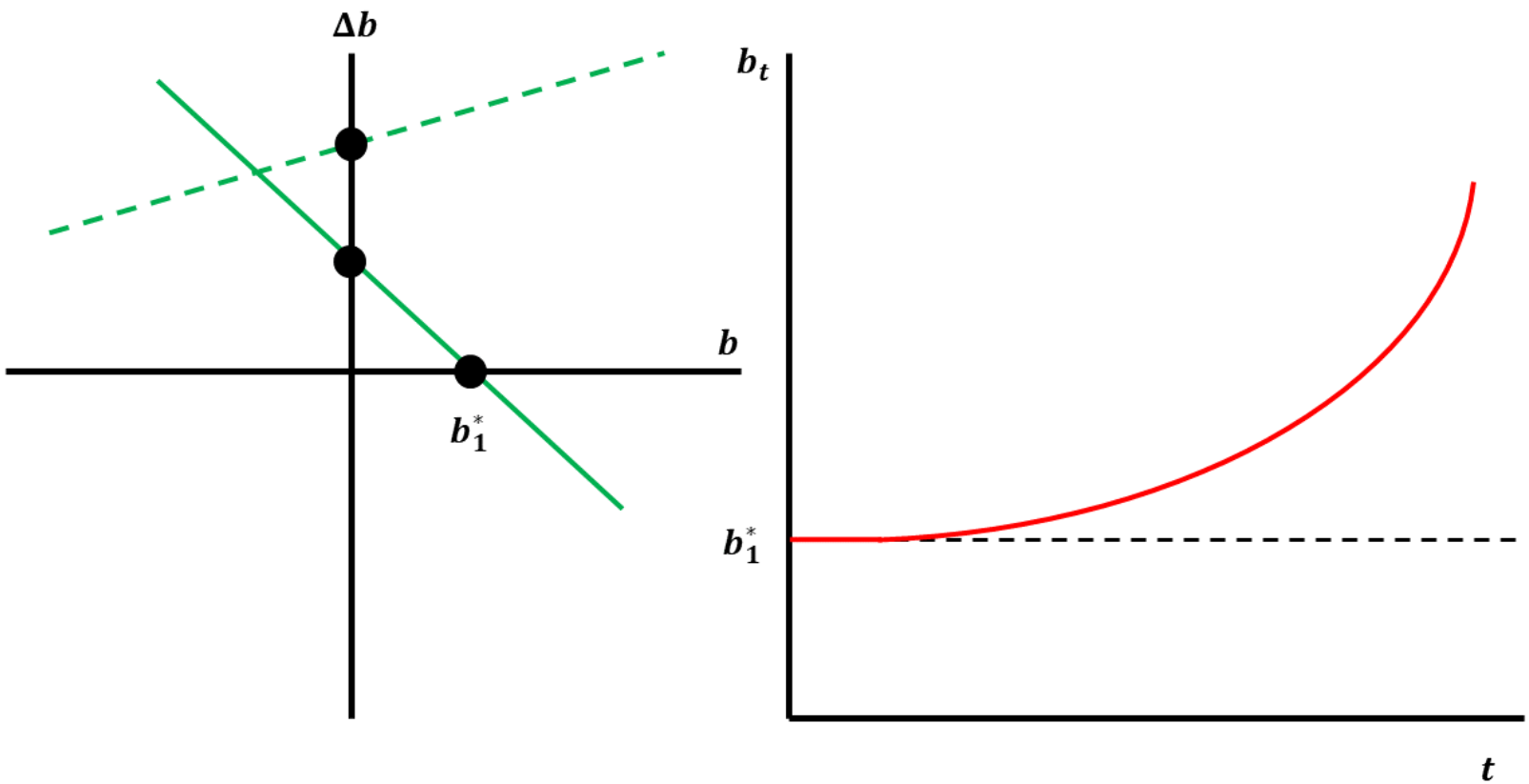
Phase Diagram and Time Path



Question 1 - Debt Crisis in Accounting Model

- If growth falls to extent that $r > g$, explosive pattern with non-convergent steady state: a debt crisis.

Phase Diagram and Time Path

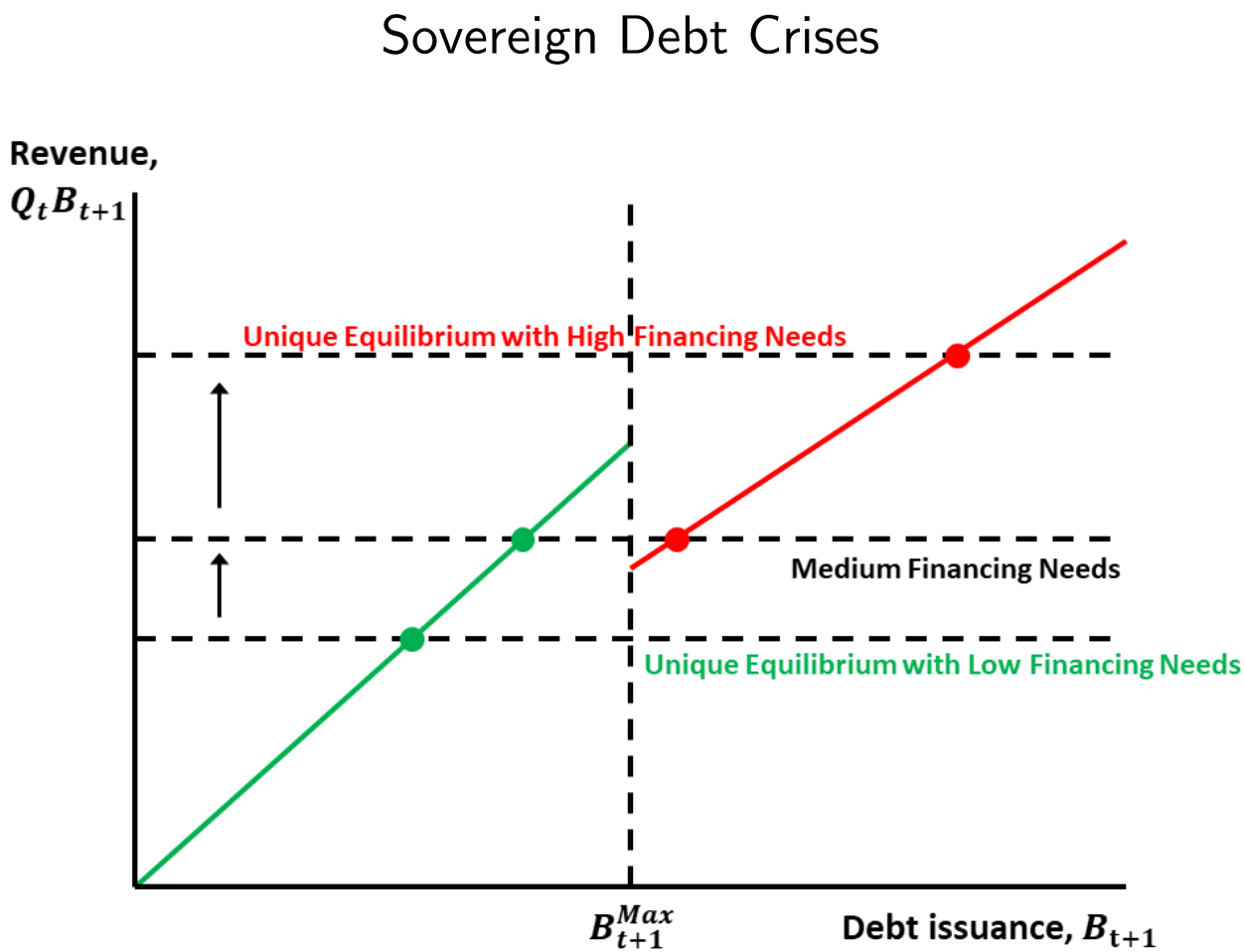


Question 1 - Sovereign Debt Crises Intuition

- For debt default, the key issue here is the **magnitude** of the increases in government financing needs.
- If current financing needs remain **low** we are unlikely to see debt crises play out.
- If **medium**, probability of crisis depends critically upon investor perceptions of debt sustainability. Potential role for IF or CB intervention.
- If **high** debt crises are more likely, but the CB / IF no incentive to intervene (debt is unsustainable).
- Also depends upon path of the virus / initial starting point for debt (we could think of example countries...) etc.

Question 1 - Sovereign Debt Crises Graph

- Change in Financing Needs is critical.



Question 2: Growth & Business Cycles.

Question 2 - Set Up

- *Figure 1 shows the natural logarithm of real output per worker (Figure 1(a)) and the total factor productivity (TFP) (Figure 1(b)) in the United Kingdom from 1950 to 2017. Figure 2 contains the (Hodrick-Prescott) cyclical component of the logarithm of real output per worker (Figure 2(a)) and TFP (Figure 2(b)) for the same period. Against this background, critically assess the role of TFP for economic fluctuations and economic growth.*

- Classic essay question.

- Is this question on **multiple** topics or **just one**?

- Take in two parts. Firstly discuss growth literature, then turn to business cycles.

Question 2 - Economic Growth

- Textbook theoretical economic growth model, Solow-Swan (1956), gives in $g_Y = g + n$ and $g_y = g$. TFP is everything!
- Qualitatively, model fits well with conditional convergence based upon s, n, g, δ, α. Mankiw, Romer & Weil (1992).
- Quantitatively, s and n have **larger** impact on g_y than model predicts, and **faster** convergence speed than seen in data.
- But TFP does not **explain** growth, it's just a residual.
- Later models make TFP **endogenous**, e.g. Romer (1990) via R&D: $g = Ba_L^{\lambda} L^{\lambda}$. TFP is still everything (but now explained).

Question 2 - Business Cycles

- Burns & Mitchell (1946) investigate higher frequencies.
- Stylized facts: RBC theory, Kyland and Prescott (1982), claims TFP also explains business cycles, matching cyclicality, coincidence and volatility of most real variables.
- Propagates via intertemporal consumption/investment decision; labour decisions; financial mechanisms.
- But concerns over **how** this is done. E.g. Summers (1986) doubted intertemporal channel is as strong as model implies.
- Big problem: **monetary shocks** also appear important and justify policy intervention (floating Walrasian equilibrium).

Question 2 - Business Cycles - Model Development

- Shocks to: technology; monetary and fiscal policy; weather and natural disasters; COVID; political; expectations.
- Models couldn't explain price movements (including r and w)
  - **Solution**: NK models developed which add **nominal rigidities** and **imperfect competition**.
- Models couldn't explain 2007/8 crisis.
  - **Solution**: Add financial intermediation block (banks).
- Models couldn't explain distributional impacts.
  - **Solution**: Add household heterogeniety "Third generation" HANK goes one step further Kaplan et al. (2018).
- Models can't explain COVID crisis.
  - **Solution**: Add SIR/epidemiology block. E.g. Kaplan et al. (2020), Guerrieri et al (2020), Eichenbaum et al. (2020).

Question 3: Business Cycles.

Question 3 - Set Up I

- *Consider the following two-period economy proposed by Mankiw and Weinzierl. The representative household lives for two periods and derives lifetime discounted utility from consumption in both periods (C_1 and C_2). The sole source of income of the household is the profit stream associated with ownership of firms. Firms maximize the present discounted value of profits. Production technology is given by $Y_t = A_t K_t$ for $t = 1, 2$, where Y_t is output, A_t is productivity and capital K_t is the only factor of production. Firms are endowed with the capital stock K_1 in period 1 and choose K_2 through their investment decision I_1 in the first period. Capital fully depreciates every period so that $K_2 = I_1$. Households are required to hold money to purchase consumption goods (i.e. there is a cash-in-advance constraint) and equilibrium in the money market requires $M_t = P_t C_t$ for $t = 1, 2$, where M_t is the money supply, and P_t is the price level. Prices are fixed in the short run ($t = 1$) and flexible in the long run ($t = 2$).*

Question 3 - Set Up II

- *continued ... The central bank has two policy tools at its disposal, the short-term nominal interest rate i_1, and the long-term money supply M_2. Further assume that preferences are represented by a constant relative risk aversion (CRRA) utility function with elasticity of substitution $\sigma < 1$. Then the equilibrium solution of this economy is given by:*

$$C_1 = \left(\frac{1}{\beta A_2}\right)^{\sigma} A_2 \frac{M_2}{(1+i_1)P_1}, \qquad C_2 = A_2 \frac{M_2}{(1+i_1)P_1},$$

$$Y_1 = \left(1 + \left(\frac{1}{\beta A_2}\right)^{\sigma} A_2\right) \frac{M_2}{(1+i_1)P_1}, \qquad I_1 = K_2 = \frac{M_2}{(1+i_1)P_1},$$

$$Y_2 = A_2 \frac{M_2}{(1+i_1)P_1}, \qquad P_2 = \frac{(1+i_1)}{A_2} P_1.$$

Question 3 (a) - Expected Fall in Future Productivity

- *(a) Analyze the dynamics of consumption, output and investment when in period* 1 *the expected future productivity of firms* (A_2) *declines.*

- System is already solved, so can just differentiate:

$$\frac{dC_1}{dA_2} = \frac{dY_1}{dA_2} = (1-\sigma)\Big(\frac{1}{\beta A_2}\Big)^{\sigma} \frac{M_2}{(1+i_1)P_1} > 0,$$

$$\frac{dI_1}{dA_2} = 0,$$

$$\frac{dC_2}{dA_2} = \frac{dY_2}{dA_2} = \frac{M_2}{(1+i_1)P_1} > 0.$$

Question 3 (a) - Intuition

- $A_2 \downarrow$ causes lower future household income $\downarrow$.
- Household consumption depends on **lifetime** income, and **smoothing** motive causes $C_1 \downarrow$ and $C_2 \downarrow$, as $\sigma < 1$ and IE dominates SE.
- Firms choose K_2 to maximise expected discounted profits:

$$\max_{K_2} P_1(A_1K_1 - K_2) + \frac{P_2A_2K_2}{1+i_1} \qquad \rightarrow \qquad P_2 = \frac{P_1(1+i_1)}{A_2}.$$

- $A_2 \downarrow$ causes $MPK_2 \downarrow$, but **offset** as lower supply causes $P_2 \uparrow$, and hence $\frac{dI_1}{dA_2} = 0$.

Question 3 (b) - Monetary Policy Offset

- *(b) Explain whether there is there scope for monetary policy to stabilize the economy and, if so, derive the nominal interest rate i_1 that would restore full capacity in period 1. Suggest an alternative policy in case the economy faces a zero lower bound on the nominal interest rate i_1.*

- Prices are **sticky** in the short run, and may not adjust to ensure full resource utilisation. Hence $Y_1 < A_1 K_1$ is possible.

- Monetary policy makers could $i_1 \downarrow$, or if at ELB $M_2 \uparrow$, to recover full resource utilisation by raising consumption (and investment) and hence output once more:

$$\frac{d^2 C_1}{dA_2 d(1+i_1)} = \frac{d^2 Y_1}{dA_2 d(1+i_1)} = -(1-\sigma)\left(\frac{1}{\beta A_2}\right)^{\sigma} \frac{M_2}{(1+i_1)^2 P_1} < 0,$$

$$\frac{d^2 C_1}{dA_2 dM_2} = \frac{d^2 Y_1}{dA_2 dM_2} = (1-\sigma)\left(\frac{1}{\beta A_2}\right)^{\sigma} \frac{1}{(1+i_1) P_1} > 0.$$

Question 3 (b) - Monetary Policy Offset

- In **flexible** price solution $C \downarrow$ with $I \uparrow$ to restore $Y_1 = A_1 K_1$. Here we use monetary policy to $C_1 \uparrow$ and $I_1 \uparrow$ relative to their **sticky** price solution to ensure full resource utilisation.
- Use $Y_1 = A_1 K_1$ in the equilibrium level of Y_1 to find the **efficient** monetary policy stance for full resource allocation.

$$A_1 K_1 = Y_1 = \left(1 + \left(\frac{1}{\beta A_2}\right)^{\sigma} A_2\right) \frac{M_2}{(1+i_1)P_1},$$

$$\frac{M_2}{(1+i_1)} = \frac{P_1 A_1 K_1}{1 + \beta^{-\sigma} A_2^{1-\sigma}},$$

$$\frac{d(1+i_1)}{dA_2} = (1-\sigma)\frac{\beta^{-\sigma} A_2^{1-\sigma}}{P_1 A_1 K_1} M_2 > 0,$$

$$\frac{dM_2}{dA_2} = -(1-\sigma)\frac{\beta^{-\sigma} A_2^{-\sigma} P_1 A_1 K_1}{(1 + \beta^{-\sigma} A_2^{1-\sigma})^2}(1+i_1) < 0.$$

- Hence if $A_2 \downarrow$, **optimal** to either $i_1 \downarrow$ or $M_2 \uparrow$ (or both).

Question 3 (b) - Monetary Policy Offset - Intuition

- Under **flexible prices** when $A_2 \downarrow$ this causes $P_1 \downarrow$.
- With P_1 fixed, prices are stuck "too high" relative to optimal.
- Real demand is "too low" at these prices, with $Y_1 < A_1 K_1$.
- CB can boost real demand, $C_1 + I_1$, by increasing the **relative nominal value** of current spending.
- CB can make current prices "relatively low" if future nominal variables expand $M_2 \uparrow$; or only a low nominal return is available when engaging in trade between periods $1 + i_1 \downarrow$.

Question 3 (c) - Great Recession

- *(c) Using the analysis above, discuss monetary policy actions in response to the Great Recession, paying particular attention to the role of unconventional monetary policy.*

- Perhaps consider the great recession as a fall in productivity.

- With low rates unconventional policy enacted:

  - Negative interest rates (now it's the ELB not ZLB!).

  - Mixed evidence on forward guidance. Success in EA "whatever it takes"; while less so in UK "unreliable boyfriend" and US "taper tantrum".

  - LSAPs & credit easing.

- Still at the ELB today (secular stagnation).

Question 4: International.

Question 4 - Set Up I

- *Consider first a small open economy which operates for two periods only,* 1 *and* 2*. Population is normalised to* 1*, and individuals have identical preferences:*

$$\ln c_1 + \beta \ln c_2, \qquad \beta \in (0, 1),$$

where c_t *denotes consumption of the output good in period-*t*. In period 1, output is exogenously given by* y_1*. The economy is financially open: residents can buy or sell an international bond* b_1*, which yields the world gross real interest rate,* $1 + r = R^{World}$*. Assume initial net foreign wealth is zero* ($b_0 = 0$)*.*

Question 4 - Set Up II

- *continued... In addition, in period* 1 *residents can invest in domestic projects* i_1*, which yield output* y_2 *in period* 2 *with decreasing returns:*

$$y_2 = A(\theta) i_1^{\alpha}, \qquad \alpha \in (0, 1),$$

where productivity $A(\theta)$ *is decreasing in barriers to trade* θ *imposed in period* 2*. Investing in project* i_1 *requires a specialised imported input with price* p *units of output. The cost of investment for a firm equals* $p(1-\tau)i_1$ *units of output, where* τ *denotes the rate at which investment is subsidised by the government. The government raises the required revenue by imposing a lump sum tax* T_1 *on residents in period* 1*. The government budget is balanced, so* $T_1 = p\tau i_1$*. Assume initially that there is no subsidy on investment* $(\tau = 0)$*.*

Question 4 (a) - IPPF-FA Derivation

- *(a) Derive the intertemporal production possibilities frontier of available consumption bundles under financial autarky. Explain how it is affected by an increase in barriers to trade θ in period 2.*

- IPPF-FA is the set of intertemporal consumption options $\{c_1; c_2\}$ obtained by varying investment, without bonds (FA).

- Budget constraints under Financial Autarky (FA):

$$c_1 = y_1 - p(1-\tau)i_1 - T_1,$$
$$c_2 = A(\theta)i_1^{\alpha}.$$

- Together these infer an IPPF:

$$c_2 = A(\theta)\left[\frac{y_1 - c_1 - T_1}{p(1-\tau)}\right]^{\alpha}.$$

Question 4 (a) - IPPF-FA Properties

- IPPF-FA is concave as:

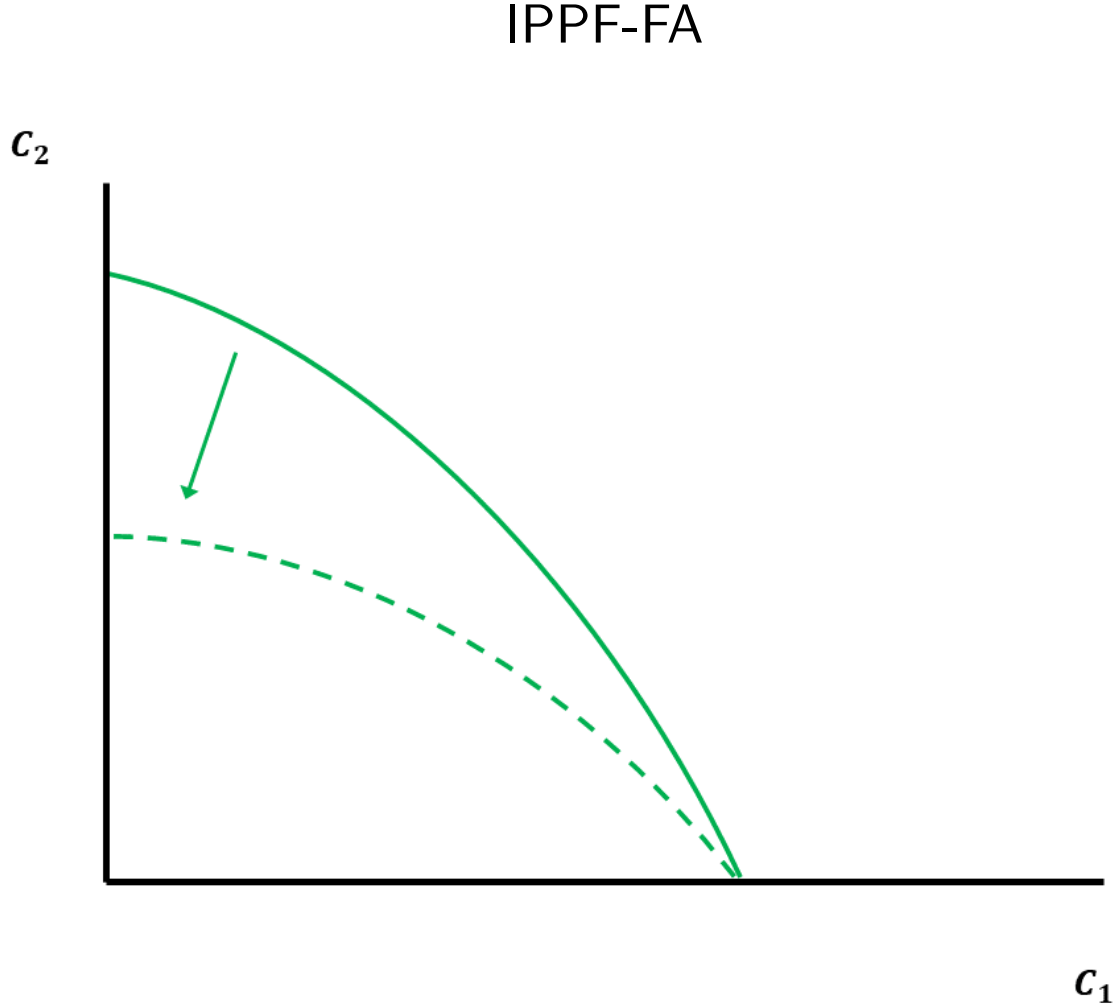
$$\frac{dc_2}{dc_1} = -\frac{\alpha A(\theta)}{p(1-\tau)}\left[\frac{y_1 - c_1 - T_1}{p(1-\tau)}\right]^{\alpha-1} < 0,$$
$$\frac{d^2c_2}{dc_1^2} = \frac{(1-\alpha)\alpha A(\theta)}{p^2(1-\tau)^2}\left[\frac{y_1 - c_1 - T_1}{p(1-\tau)}\right]^{\alpha-2} > 0.$$

- IPPF-FA will rotate inwards if $\theta \uparrow$, as told $\frac{dA(\theta)}{d\theta} < 0$:

$$\frac{dc_2}{d\theta} = \frac{dA(\theta)}{d\theta}\left[\frac{y_1 - c_1 - T_1}{p(1-\tau)}\right]^{\alpha} < 0.$$

Question 4 (a) - IPPF-FA Graph

- Rotates inwards if $\theta \uparrow$.



Question 4 (b) - Additional Trade Barriers - Maths I

- *(b) Suppose that initially the gross real interest rate under financial autarky satisfies $R^{FA} = R^{World}$. Analyze how an increase in trade barriers θ would affect the country's current account in period 1.*

- Solve the model:

$$\max_{b_1, i_1} \ln[y_1 - p(1-\tau)i_1 - T_1 - b_1] + \beta \ln[A(\theta)i_1^{\alpha} + (1+r)b_1],$$

- Two first order conditions and two budget constraints:

$$\begin{aligned} c_2 &= \beta(1+r)c_1, \\ p(1-\tau)c_2 &= \alpha\beta A(\theta) i_1^{\alpha-1} c_1, \\ b_1 + c_1 &= y_1 - p(1-\tau)i_1 - T_1, \\ c_2 &= A(\theta)i_1^{\alpha} + (1+r)b_1. \end{aligned}$$

Question 4 (b) - Additional Trade Barriers - Maths II

- In the trading equilibrium r is exogenous, hence solve two FOCs to give:

$$i_1 = \left[\frac{\alpha\beta A(\theta)}{p(1-\tau)\beta(1+r)}\right]^{\frac{1}{1-\alpha}}.$$

- IBC then combined with Euler Equation (for bonds) gives:

$$c_1 + \frac{c_2}{1+r} = y_1 - p(1-\tau)i_1 - T_1 + \frac{A(\theta)i_1^{\alpha}}{1+r},$$

$$c_1 = \frac{1}{1+\beta}\left[y_1 - p(1-\tau)i_1 - T_1 + \frac{A(\theta)i_1^{\alpha}}{1+r}\right].$$

- **Recursively** solving the rest of the model:

$$c_2 = \beta(1+r)c_1,$$

$$b_1 = y_1 - p(1-\tau)i_1 - T_1 - c_1.$$

Question 4 (b) - Additional Trade Barriers - Maths III

- We are particularly interested in the current account:

$$\begin{aligned} b_1 &= y_1 - p(1-\tau)i_1 - T_1 - c_1, \\ &= \frac{1}{1+\beta}\left[\beta(y_1 - T_1) - \beta p(1-\tau)i_1 - \frac{A(\theta)i_1^{\alpha}}{1+r}\right]. \end{aligned}$$

- Hence impact of change in θ is:

$$\frac{db_1}{d\theta} = -\frac{1}{1+\beta}\left[\left(\beta p(1-\tau) + \frac{\alpha A(\theta) i_1^{\alpha-1}}{1+r}\right)\frac{di_1}{d\theta} + \frac{i_1^{\alpha}}{1+r}\frac{dA(\theta)}{d\theta}\right].$$

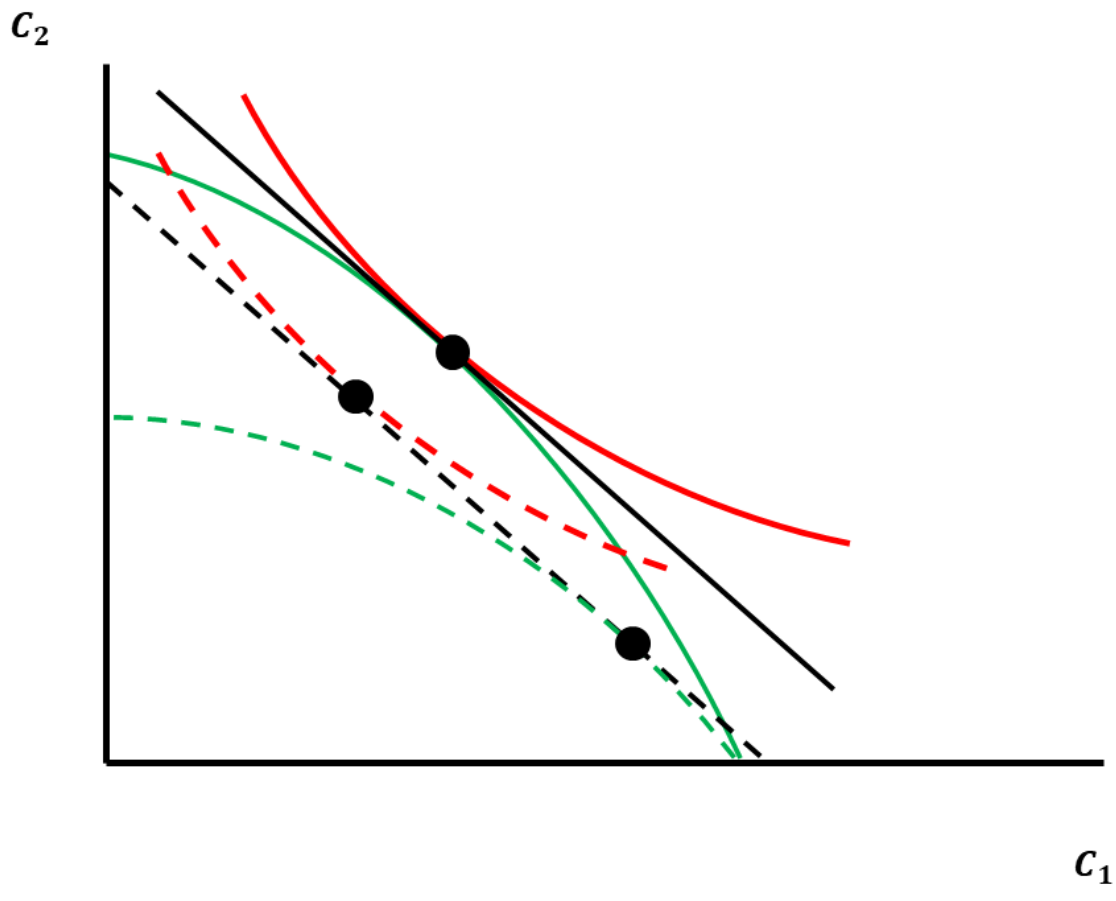
- This expression is **positive** since we are told $\frac{dA(\theta)}{d\theta} < 0$ and:

$$\frac{di_1}{d\theta} = \frac{\alpha\beta}{p(1-\tau)(1-\alpha)\beta(1+r)}\left[\frac{\alpha\beta A(\theta)}{p(1-\tau)\beta(1+r)}\right]^{\frac{\alpha}{1-\alpha}}\frac{dA(\theta)}{d\theta} < 0.$$

Question 4 (b) - Additional Trade Barriers - Graph

- Trading and *FA* allocations initially equal.
- When $\theta \uparrow$, must move to right (due to concavity and rotation inwards of IPPF-FA). Hence now $c_1 > c_1^{FA}$, with $b_1 > 0$.

IPPF-FA



Question 4 (b) - Additional Trade Barriers - Intuition

- When trade barriers, θ, increase this makes it **more expensive** to physically transform goods from today to tomorrow via investment.
- Utility **falls**.
- Given initial starting point, now relatively well endowed with current income.
- At world prices can **smooth** consumption by saving income today through $b_1 > 0$ and consuming tomorrow.
- **Separation** of production and consumption points.

Question 4 (c) - Investment Subsidy - IPPF-FA

- *In the current debate on industrial policy, some proposals entertain the idea of subsidising firms, to counteract any potential negative effects of higher trade barriers on investment.*

- *(c) Evaluate this by analyzing how a positive investment subsidy* $\tau \in (0, 1)$ *affects the intertemporal production possibilities frontier of available consumption bundles under financial autarky, and the current account.*

- It turns out this is complicated!

Question 4 (c) - Investment Subsidy - IPPF-FA

- Budget constraints under FA:

$$c_1 = y_1 - p(1-\tau)i_1 - T_1,$$
$$c_2 = A(\theta)i_1^{\alpha}.$$

- Firms/HHs know $T_1 > 0$ but **ignore** the aggregate condition:

$$T_1 = p\tau i_1.$$

- IPPF-FA is set of options that Firms/HHs generate varying i_1.

- Horizontal shift **inwards** as when $i_1 = 0$, this implies $c_1 = y_1 - T_1$ with $c_2 = 0$.

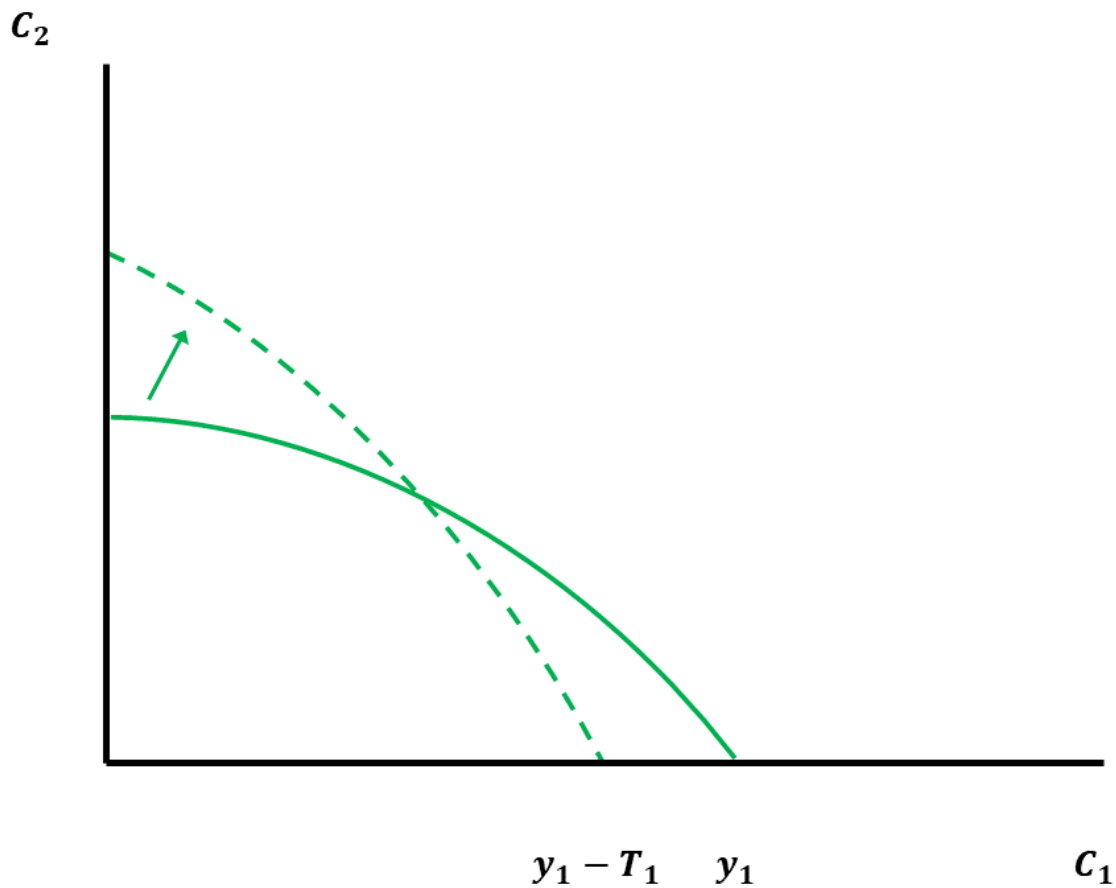
- Also **steeper** slope (ignoring general equilibrium effect $\frac{dT_1}{d\tau}$):

$$\frac{\partial c_2}{\partial \tau} = \frac{\alpha A(\theta)}{1-\tau}\left[\frac{c_1 - y_1 + T_1}{p(1-\tau)}\right]^{\alpha} > 0.$$

Question 4 (c) - Investment Subsidy - Graph

- Inwards shift.
- Higher gradient.

IPPF-FA



Question 4 (c) - Investment Subsidy - Impact on Welfare

- From above, our (recursive) general equilibrium system:

$$
\begin{aligned}
i_1 &= \left[\frac{\alpha\beta A(\theta)}{p(1-\tau)\beta(1+r)}\right]^{\frac{1}{1-\alpha}}, \\
c_1 &= \frac{1}{1+\beta}\left[y_1 - p(1-\tau)i_1 - T_1 + \frac{A(\theta)i_1^{\alpha}}{1+r}\right], \\
c_2 &= \beta(1+r)c_1, \\
b_1 &= y_1 - p(1-\tau)i_1 - T_1 - c_1.
\end{aligned}
$$

- Use GE relationship $T_1 = p\tau i_1$ and simplify:

$$
\begin{aligned}
i_1 &= \left[\frac{\alpha\beta A(\theta)}{p(1-\tau)\beta(1+r)}\right]^{\frac{1}{1-\alpha}}, \\
c_1 &= \frac{1}{1+\beta}\left[y_1 - pi_1 + \frac{A(\theta)i_1^{\alpha}}{1+r}\right], \\
c_2 &= \beta(1+r)c_1, \\
b_1 &= \frac{1}{1+\beta}\left[\beta(y_1 - pi_1) - \frac{A(\theta)i_1^{\alpha}}{1+r}\right].
\end{aligned}
$$

Question 4 (c) - Investment Subsidy - Differentials

- Differentiate the system:

$$\frac{di_1}{d\tau} = \frac{1}{(1-\alpha)(1-\tau)}\left[\frac{\alpha\beta A(\theta)}{p(1-\tau)\beta(1+r)}\right]^{\frac{1}{1-\alpha}} > 0,$$

$$\frac{dc_1}{d\tau} = \frac{1}{1+\beta}\left[\alpha\frac{A(\theta){i_1}^{\alpha-1}}{1+r} - p\right]\frac{di_1}{d\tau} = -\frac{p\tau}{1+\beta}\frac{di_1}{d\tau} < 0,$$

$$\frac{dc_2}{d\tau} = \beta(1+r)\frac{dc_1}{d\tau} < 0,$$

$$b_1 = -\frac{1}{1+\beta}\left[\beta p + \frac{\alpha A(\theta) i_1^{\alpha-1}}{1+r}\right]\frac{di_1}{d\tau} < 0.$$

- i_1 **increasing** in subsidy. Unsurprising: if subsidised do more!
- b_1 (alternative/substitute way to transform consumption between periods) is **decreasing** in the subsidy.
- c_1 and c_2 (and hence utility) **decreasing** in subsidy. Proportional subsidy with lump sum tax distorts allocations introducing welfare wedge. Mention welfare theorems.

Question 4 (d) - Metzler Diagram

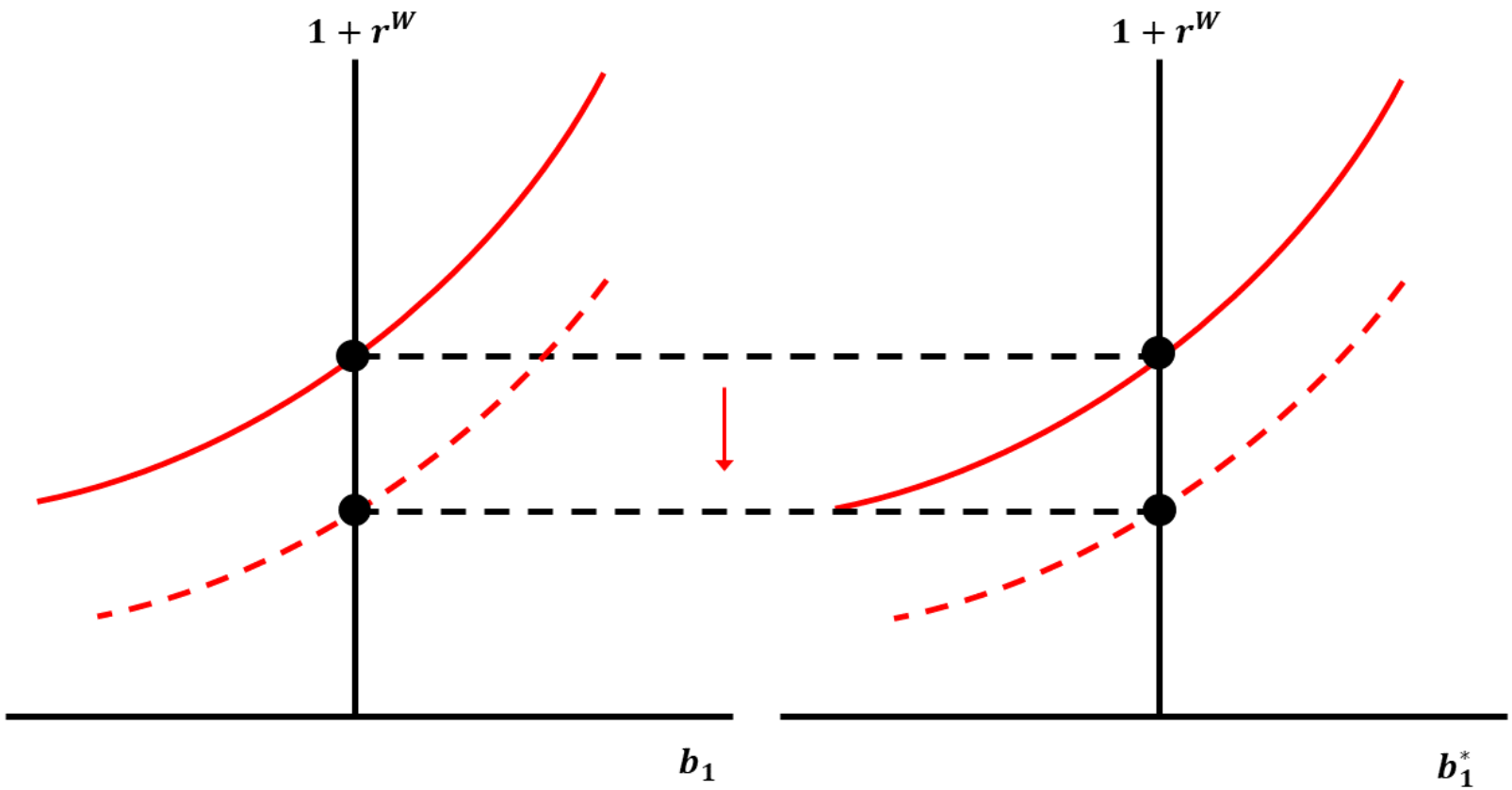
- *So far, we have considered a single small open economy. Now suppose that a trade war negatively affects productivity in all countries.(d) Using a Metzler diagram, analyze the global effects of a trade war if greater trade barriers lower productivity symmetrically across all countries. Discuss how a trade war could affect current account imbalances.*

- Both countries shift downwards.

Question 4 (d) - Metzler Diagram

- Assuming **symmetric** countries, both shift downwards. $1 + r^W$ falls. Could consider **asymmetric** results too.

Metzler Diagram



Your Questions.

Question 1 - International - Metzler Diagram I

- *PS7: What is the intuition behind why Foreign country has a steeper Metzler curve?*

- Optimal allocation is determined by Euler Equation.

$$u'(C_1) = \beta \mathbb{E}[u'(C_2)]R.$$

- Use BCs to find equilibrium relationship between B_1 and R:

$$u'(Y_1 - B_1) = \beta \mathbb{E}[u'(Y_2 + RB_1)]R.$$

- **Implicitly** differentiate to find (inverse) slope of Metzler, $\frac{dB_1}{dR}$:

$$-u''(C_1)dB_1 = \beta \mathbb{E}[u'(C_2)dR + u''(C_2)RB_1 dR + u''(C_2)R^2 dB_1],$$

$$\frac{dB_1}{dR} = -\frac{\mathbb{E}[\beta u'(C_2) + \beta u''(C_2)RB_1]}{\mathbb{E}[u''(C_1) + \beta u''(C_2)R^2]}.$$

- Intuition very unclear!

Question 1 - International - Metzler Diagram II

- In general, nothing more we can say.
- In specific case of no uncertainty, and CRRA we utility may rewrite the above as:

$$\frac{dB_1}{dR} = \frac{\frac{\sigma C_2}{R} - B_1}{R(1+\beta^\sigma R^{\sigma-1})} = \frac{\frac{\sigma C_2}{R} - (Y_1 - C_1)}{R(1+\beta^\sigma R^{\sigma-1})}.$$

- Overall impact depends upon two effects:
  1. SE. As $R \uparrow$ future consumption more attractive, $C_2 \uparrow$, while $C_1 \downarrow$ and $B_1 \uparrow$. Strength governed by $\frac{\sigma C_2}{R}$ term.
  2. IE. As $R \uparrow$ borrowers pay more and hence will lower lifetime income. $C_1 \downarrow$ and $B_1 \uparrow$. Strength governed by B_1 term.
- Inadvisable to focus upon risk properties in this case.

Question 2 - Business Cycles

- *When households are risk averse and technical persistence exists ($\mu > 0$) are both the endogenous consumption and capitals switched on?*

- **Yes**. Whenever $\mu > 0$ **future** MPK changes along with current, so an endogenous capital channel of persistence.

- If risk averse, will still be a separate, distinct **consumption smoothing** motive.

Question 3 - Business Cycles

- *PS5: How to we get to the final ARMA(1,1) process for y_t?*
- We have the system:

$$z_t = \mu z_{t-1} + \psi_t,$$
$$y_t = z_t + \alpha\bar{k} + \frac{\alpha\mu}{1-\alpha} z_{t-1}.$$

- To **solve** the system we must eliminate the **endogenous** variable $z_t, z_{t-1}, \cdots$ and leave $y_t, y_{t-1}, \cdots$ in terms of only **exogenous** shocks $\psi_t, \psi_{t-1}, \cdots$, hence:

$$y_t = z_t + \alpha\bar{k} + \frac{\alpha\mu}{1-\alpha} z_{t-1},$$
$$(-) \quad \mu y_{t-1} = \mu z_{t-1} + \alpha\mu\bar{k} + \frac{\alpha\mu^2}{1-\alpha} z_{t-2}.$$
$$y_t - \mu y_{t-1} = \psi_t + \alpha(1-\mu)\bar{k} + \frac{\alpha\mu}{1-\alpha}\psi_{t-1}.$$

- By inspection this is of ARMA(1,1) form:
$A_t - \alpha A_{t-1} = \gamma + B_t - \beta B_{t-1}$, with $B_t \sim N(0, \sigma^2)$.

Question 4 - Business Cycles: Utility

- What is the intuition behind σ in:

$$U(C_0, \ell_0) = \frac{C_0^{1-\sigma}}{1-\sigma} + \gamma \frac{(1-\ell_0)^{1-\sigma}}{1-\sigma}.$$

- Actually, to be well defined:

$$U(C_0, \ell_0) = \frac{C_0^{1-\sigma} - 1}{1-\sigma} + \gamma \frac{(1-\ell_0)^{1-\sigma} - 1}{1-\sigma}.$$

- Differentiate:

$$U'_C = C_0^{-\sigma} \quad U''_C = -\sigma C_0^{-\sigma-1}.$$

- Put together:

$$\text{Coef. of RRA} \equiv -\frac{U''_C}{U'_C} C = -\frac{-\sigma C_0^{-\sigma-1}}{C_0^{-\sigma}} C_0 = \sigma.$$

Question 5 - Business Cycles: Investment

- RBC models have a single representative household. How does borrowing work?

- As always, the equilibrium condition for household saving is:

$$S(r) = I(r).$$

- Remember, this equilibrium condition simply determines the price, r.

- Without capital, we would simply write:

$$S(r) = 0.$$

Question 6 - Business Cycles: Welfare

- What is meant by the asymmetric welfare effects in NK model.
- This is complicated! ... and usually ignored.
- Distortion 1: Imperfect competition (P too high, Y^n too low).
- Distortion 2: Sticky prices (sluggish response with $Y_t \neq Y^n$).
- Output: actual, Y_t; natural (flexible) level, Y^n; and efficient (welfare maximising) level, Y^*.
- If $Y^n < Y^*$, a shock that moves $Y_t > Y^n$, moves economy closer to Y^*.

Question 7 - Policy: Efficient Policy Frontier

- What makes the Efficient Policy Frontier (EPF) a long run concept?
- Policy targets the **current level** of variables (e.g. π_t).
- Taking into account today's shocks (e.g. η_t, and ε_t).
- But the EPF exists in **variances** (e.g. σ_π^2):

$$\sigma_\pi^2 = \left(\sqrt{\sigma_\varepsilon^2} - \theta\sqrt{\sigma_Y^2}\right)^2.$$

- So we need to observe **multiple shocks** to trace it out.

Question 8 - Policy: Tax Smoothing

- What is the intuition behind the tax rate smoothing result?
- We assume policymakers face **convex** losses today and tomorrow in tax rates.
- This optimally implies **smoothing**, with τ_1 and τ_2 jointly determined.
- To fund a given level of government spending it's better to tax a little in both periods than all in one.
- This means better to have small losses in each argument of loss function than the same loss in one argument.
- If the market price moving loss between periods is equal to the rate of preference for doing so, then **perfect smoothing**.

Revision: Some Guidance.

Revision: Ahead of Time

- Create document (practise method for diagrams).
- Add all literature references in bibliography (group by topic).
- Your "known knowns"
  - Have you already done that question?
- Practise open book essays using previous exams.

Revision: During the Exam

- Give all questions a chance.
- Determine if questions are on **multiple** topics or **just one**.
- Determine your answer approach:
  - Build a model.
  - Graphical explanation.
  - Critical essay.
- You **must** plan essays (start with base essay, add detail).
- You **must** read over and check your full answer.

Revision: What to Avoid

- Factual mistakes (including in literature).
- Not understanding the **main** point of question.
- Extensive use of lecture material (a false friend).
- Long derivations that go nowhere.

Revision: Keep in Mind

- Like regular exam answers, essays should be self-contained.
- Sufficient detail should be provided to understand your points.
- The more precise and informative the essay is in addressing the question, the higher the mark.

Final Thoughts

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- How to prepare for open book exams.
- Good luck!